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# Practical Basic Programs

edited by  
Lon Poole

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**PRACTICAL BASIC PROGRAMS**

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## Preface

We collected these programs together in this book to address the continuing need for readily available and easy-to-use computer programs that do something useful. The supply of such programs has not kept pace with the demand. On one hand, the number of computer users is growing at an astounding rate, thanks chiefly to the availability of inexpensive small computers. An increasing number of these people, many of them first-time users, are interested only in the practical aspects of computing. Today, those who view the computer solely as a means of entertainment are few and far between. While more practical programs are now available, many contributed by new users, there just aren't enough. And those that do exist are hard to find. So we brought together in this book forty relatively short programs covering a wide range of practical applications.

### Acknowledgments

Steven Cook, Martin McNiff, and Robert Thomson conceived, designed, and wrote many of these programs. Collectively they tested all of these programs and prepared the final write-ups and program listings for publication.

Dr. Samuel H. Westerman provided the concepts, designs, and initial program listings for eighteen of these programs: Income Averaging, Continuous Interest Compounding, Depreciation Switch, Apportionment by Ratios, Profit Sharing Contributions, Statistical Estimation Theory, Statistics, Unbiased Estimator of Standard Deviation, Chi-Square, Data Forecasting Divergence, Newtonian Interpolation, Lagrangian Interpolation, Sums of Powers, Factorials, Temperature Conversion, and Musical Transposition. He also provided source material for the write-ups for the eighteen programs.

Richard E. Beckwith, Ph.D. provided the concept, design, program code, and write-up for the program Swedish Machine (Queuing Theory).

George M. Blake suggested programs Accrued Interest on Bonds and Current Value of a Treasury Bill.





# Introduction

## Purpose

Considering all the small computers people have bought in recent years, it should be easy to find practical computer programs. This is especially true since few users still consider their computer just a diversion. But practical programs are not readily available. The purpose of this book is to help fill that void. All forty programs in this book are useful computer applications. The Basic program listings are included. Type them into your computer and they are ready to run. Both the programmer and the non-programmer alike benefit from this; neither one has any programming to do. All of which saves everyone time; the non-programmer needn't learn programming and the programmer has more time to write programs no one else has written.

While you don't have to be a programmer to use this book, you must understand the subject matter of the programs you wish to use. It is beyond the scope of this book to explain how, when, where, or why you would use any of them. This does not mean you must be a tax accountant in order to use the Income Averaging program, or a management science professional to use the Transportation Algorithm program. There are sample runs and practice problems for each program. Chances are you can figure out the program's applications from them. And if you understand the applications to some extent, but would like more information, you will find further reading suggested in the References section of many programs.

This book has a secondary purpose as well, and that is to show by example the wide range of subjects that lend themselves to computerization. All too often, computer users who have cut their teeth on entertainment computing have trouble coming up with ideas for practical computing. So even if you don't see a program in this book that is exactly what you need, you may find it easier to invent your own practical applications after studying some of these.

As you look through the programs in this book, you may discover that you can use pieces of the programs or some of the programming techniques in your own work. For example, embodied in these programs is a function for rounding arithmetic calculations to the nearest cent and a subroutine for pausing at the end of each full display screen. For that matter you may be able to use an entire program as a component part of your own larger, more complex program. Some of these programs themselves make use of programs from the book *Some Common Basic Programs*, also published by OSBORNE/McGraw-Hill.

## Organization

These programs find their primary applications in four general areas: financial, management decision, statistics, and mathematics and science. This arbitrary classification has no bearing on the utility of the programs per se. Clearly, the real question is not what label we have applied to a program, but rather how it can be used.

Towards this end, each program includes a complete write-up in addition to its listing. Each write-up begins with a discussion of its subject matter, its required inputs, and its resultant output. In some cases, there are limitations in the algorithm the program employs, or in the applicability of the program. These are described next. Following this in many programs is a Program Notes section. It tells you how to make minor program changes that make the program operate in a slightly different way, accommodate more or less data, and so forth. These changes may make the difference between the program being convenient or difficult for you to use. The Program Notes section also explains any complex or tricky aspects of the way the program itself is written. Generally speaking, it addresses the technical aspects of implementing the application with a computer program.

Following this narrative material is an example of the program in use. Wherever possible, we set this example in a more or less real-life situation. An example which states a situation that can be resolved by using the program is more instructive than a list of raw data which you can plug into the program. The point of doing this is not to exercise our imagination in concocting these situations, but to exercise your imagination in visualizing potential uses of the program. The examples demonstrate as many program features as they can in a reasonably sized problem. We provide the correct answers to the unknowns of the example.



The answers may be in narrative form, or they may be an inherent part of the sample run, which comes next. The sample run shows the dialogue which occurs between the user and the computer when the program is used to answer the questions posed in the example. Compare the user's inputs and the computer's outputs in the sample run with the problem stated in the example. You should be able to determine how you would use the program to solve a similar problem.

Practice problems follow each example. Use them to gain more familiarity with different ways you can use a program. Generally, we provide only the answers to these practice problems and not sample runs.

The complete Basic program listing comes next. The listings are documented with in-line remarks. The remarks make it easier for you to figure out how the program works, if you are so inclined. The remarks (which always begin with the Basic command REM) are not essential to program operation but they will facilitate your understanding of it.

Finally, we list references for most programs. Investigate these books, articles, etc. if you wish to read more about the subject matter of the program.

## Basic Compatibility

We have tried to write these programs in a very general Basic not specific to any one computer. Our concept of a general Basic is based in part on the experience of writing *Some Common Basic Programs*, and especially on the response from readers of that book. Additionally, the contributors to this book have collectively used many different versions of Basic available on popular small computers. These include Applesoft Basic, Atari Basic, Commodore Basic, CBasic, Digital Equipment Corporation's (DEC) Basic-Plus, Radio Shack TRS-80 Basic, Texas Instruments Basic, and Wang Laboratories Basic. We adopted programming conventions that would allow the programs in this book to run unmodified on as many versions of Basic as we could. In some cases we could not do this. String variables, string functions, and string operators are treated too differently in some versions of Basic. Wherever we could anticipate such compatibility problems, we identified them in the appendix and suggested alternatives there as well.

None of these programs requires a mass storage device — disk or tape — for storing data. Thus the widely varying methods for accessing data files in Basic are not a problem with the programs in this book. Of course you will want to store the programs themselves on a tape or disk once you have typed them in. But this is a fairly straightforward procedure which is undoubtedly adequately described in your computer owner's manual.

## How to Use These Programs

Follow the steps listed below to use any of these programs.

1. Read the program write-up and familiarize yourself with how the program works. Read the cited references if they will give you a better understanding of the subject matter which the program addresses. Be sure the program does what you need it to do before going any further.

2. Type the program listing into your computer. Since the remark statements (those that begin with REM) are not essential to program operation, you need not type them in. By doing so, you will save time and programs will take less space (on most computers), and the programs may even run marginally faster. But if you plan to modify a program extensively, you may be better off including its remarks, since they can be very helpful in tracing program logic flow during debugging.

3. Check your program listing carefully for correctness. Compare it line-by-line and character-by-character with the published listing. Correct any discrepancies.

4. Save the program on tape or disk. Do it now, before you run the program. That way you can easily retrieve it in the event that anything happens while you are running it.

5. Run the example exactly as shown in the sample run. If you have done everything right to this point, the results should be very similar to those published. Your answers will differ slightly from those in the book if your computer has a different level of numerical precision than ours.

6. If your answers differ markedly from ours, or your program does not run at all (i.e., you get some sort of error message), it is time for some detective work. First, double-check and triple-check your listing against the published one. We cannot overemphasize the importance of this scrutiny. Check for missing program lines and incorrect line numbers. Make sure you have entered the right letter or digit. It is often easy to confuse zeros and O's, ones and I's, two's and Z's, fives and S's, and U's and V's.

If reviewing your listing doesn't disclose any typographical errors, check the appendix. See if any of the Basic irregularities discussed there apply to your computer. If so, apply the suggested changes to your program and rerun it.

By now, your programs should be running correctly. If not, have someone else look your program over. Often another set of eyes can see things that you will repeatedly miss. Try putting the program aside for a while and coming back to it. After a short break, errors you didn't see before may be glaringly obvious.

There is always the possibility that the program may still be incompatible with your Basic. Detecting this requires some knowledge of programming. Compare the syntax of the program from the book with your Basic syntax. Differences may be easy to spot or they may be subtle ones. Experience, ingenuity, and knowledge of your Basic's quirks are your best tools at this point.

7. As a further test of your program, run the practice problems. Compare your answers with those in the book. Here again, they should be very close, though some slight discrepancy is not at all unusual.



---

## Income Averaging

---

This program calculates U.S. federal income tax using the income averaging method (Form 1040, Schedule G). It determines whether a taxpayer qualifies for income averaging, and if so, it displays the entries to complete Schedule G. The program is based on 1979 tax forms, tax rates, and tax laws. It is devised to be used for as many years in the future as the law, rates, and forms remain the same as in 1979.

To use the program, you must enter the taxpayer's name, the taxable year, and the taxpayer's filing status that year (i.e., single, married filing jointly, married filing separately, unmarried head of household, or qualifying widow(er)). You then enter the taxpayer's base period income — the four years preceding the taxable year. For 1977 and later, this is the amount from line 34 of Form 1040, or line 11 of Form 1040A (line 10 on the 1977 and 1978 Forms 1040A). You must also enter the number of exemptions for each year 1977 and later, when the program asks for them. For any years of the four-year base period before 1977, you enter the taxable income directly. We should emphasize that you should enter an income figure — even a negative figure — for each year, and you should enter the total number of exemptions claimed each year (when requested), *even though the taxpayer had no net income or even though it was a negative taxable income*.

Note that even though Schedule G directs that line 3 may not be less than zero, whenever the Internal Revenue Service has been confronted with the legislative history of the applicable section of the Internal Revenue Code, it has backed off, and permitted a negative figure on line 3; this program takes advantage of that fact. On the other hand, note that line 6 on Schedule G may not be less than zero, and the program takes account of that, too.

The program then asks you for other applicable income amounts (e.g., excluded foreign income) and the taxable income from Schedule TC for the taxable year. It then determines whether income averaging is permissible. If so, it displays the amounts you need in order to fill out Schedule G (1979 format).

### Program Notes

The program rounds all calculations to the nearest penny. Some taxpayers prefer to work only to the nearest dollar. To put whole dollar calculations into effect, change line 40 as shown below, and when the program asks you to enter dollar amounts, enter them in whole dollars only.

```
39 REM ROUND OFF TO WHOLE DOLLARS
40 DEF FNR (X)=INT (X+.5)
```

The 1979 Schedule G reproduced below shows how the elements of array A( ) correspond to the lines and columns of Schedule G, from A(1), the taxable year in the upper right corner, to A(44), the computed tax amount. Note that variables A(4), A(5), A(8), and A(9) are in hatched boxes (the I.R.S. intends that they remain blank in 1979). For 1979, the program accounts for that by making them all zero. As years pass, the hatching will pass off to the right, and entries will be required in those boxes. For example, in 1980, the boxes in lines 1 and 2, column C, will not be hatched; A(4) and A(8) will be active if the program is run for taxable year 1980.

In a similar manner, the hatching on line 5 will encroach upon columns C and D. Thus variables A(14) and A(15) will become inactive in 1980 and 1981.



# Income Averaging

▶ See instructions on pages 2 and 3.  
▶ Attach to Form 1040.

A (1)

18

Name(s) as shown on Form 1040

Your social security number

	(a) 1st preceding base period year 1978	(b) 2d preceding base period year 1977	(c) 3rd preceding base period year 1976	(d) 4th preceding base period year 1975
<b>Base Period Income and Adjustments</b>				
1 Enter amount from: Form 1040 (1977 and 1978)—line 34 Form 1040A (1977 and 1978)—line 10 . . .	A (2)	A (3)	A (4)	A (5)
2 Multiply \$750 by the total number of exemp- tions claimed in 1977 and 1978 . . . . .	A (6)	A (7)	A (8)	A (9)
3 Taxable income (subtract line 2 from line 1) If less than zero, enter zero (see instructions) .	A (10)	A (11)	A (12)	A (13)
4 Income earned outside of the United States or within U.S. possessions and excluded un- der sections 911 and 931 . . . . .	A (16)	A (17)	A (18)	A (19)
5 On your 1979 { 2 or 5 enter \$3,200 } { in columns Form 1040, if 1 or 4 enter \$2,200 } { (c) and (d) you checked box 3 enter \$1,600 . . . . .			A (14)	A (15)
6 Base period income (add lines 3, 4 and 5) .	A (22)	A (23)	A (24)	A (25)
<b>Computation of Averageable Income</b>				
7 Taxable income for 1979 from Schedule TC (Form 1040), Part I, line 3 . . . . .	7	A (26)		
8 Certain amounts received by owner-employees subject to a penalty under sec- tion 72(m)(5) . . . . .	8	A (20)		
9 Subtract line 8 from line 7 . . . . .	9	A (27)		
10 Excess community income . . . . .	10	A (21)		
11 Adjusted taxable income (subtract line 10 from line 9). If less than zero, enter zero . . . . .	11	A (28)		
12 Add columns (a) through (d), line 6, and enter here . . . . .	12	A (29)		
13 Enter 30% of line 12 . . . . .	13	A (30)		
14 Averageable income (subtract line 13 from line 11) . . . . .	14	A (31)		

If line 14 is \$3,000 or less, do not complete the rest of  
the form. You do not qualify for income averaging.

G

<b>Computation of Tax</b>				
15 Amount from line 13 . . . . .	15	A (32)		
16 20% of line 14 . . . . .	16	A (33)		
17 Total (add lines 15 and 16) . . . . .	17	A (34)		
18 Excess community income from line 10 . . . . .	18	A (21)		
19 Total (add lines 17 and 18) . . . . .	19	A (35)		
20 Tax on amount on line 19 (see caution below) . . . . .	20	A (36)		
21 Tax on amount on line 17 (see caution below) . . . . .	21	A (37)		
22 Tax on amount on line 15 (see caution below) . . . . .	22	A (38)		
23 Subtract line 22 from line 21 . . . . .	23	A (39)		
24 Multiply the amount on line 23 by 4 . . . . .	24	A (40)		
Note: If no entry was made on line 8 above, skip lines 25 through 27 and go to line 28.				
25 Tax on amount on line 7 (see caution below) . . . . .	25	A (41)		
26 Tax on amount on line 9 (see caution below) . . . . .	26	A (42)		
27 Subtract line 26 from line 25 . . . . .	27	A (43)		
28 Tax (add lines 20, 24, and 27). Enter here and on Schedule TC (Form 1040), Part I, line 4 and check Schedule G box . . . . .	28	A (44)		

Caution: Use Tax Rate Schedule X, Y or Z from the Form 1040 instructions to figure your tax on lines 20, 21, 22, 25 and 26. Do not use the tax tables.

### Example

Uriah Heep is married and has one dependent. Line 34 of his 1978 and 1977 Forms 1040 shows \$16,000 and \$14,000. His taxable incomes for 1976 and 1975 were \$12,000 and \$10,000. He has no excluded foreign income, no amounts subject to penalty under Code Section 72(m)(5), and no excess community income. His 1979 taxable income is \$52,500. How would you use this program to help fill out his Schedule G for 1979?

Answer:

INCOME AVERAGING

TAXPAYER'S NAME IS:

? URIAH HEEP

TAXABLE YEAR:

? 1979

ENTER FILING STATUS--

--1 FOR SINGLE

--2 FOR MARRIED/JOINT

--3 FOR MARRIED/SEPERATE

--4 FOR HEAD OF HOUSEHOLD

--5 FOR QUALIFYING WIDOW(ER)

? 2

ENTER THE INCOME FIGURE CORRESPONDING

TO LINE 34 ON FORM 1040, OR ON FORM

1040A, CORRESPONDING TO LINE 11(1979)

OR LINE 10(1977-1978).....

FOR THE YEAR 1978

? 16000

HOW MANY EXEMPTIONS CLAIMED THAT YEAR?

? 3

ENTER THE INCOME FIGURE CORRESPONDING

TO LINE 34 ON FORM 1040, OR ON FORM

1040A, CORRESPONDING TO LINE 11(1979)

OR LINE 10(1977-1978).....

FOR THE YEAR 1977

? 14000

HOW MANY EXEMPTIONS CLAIMED THAT YEAR?

? 3

ENTER TAXABLE INCOME FOR YEAR 1976

? 12000

ENTER TAXABLE INCOME FOR YEAR 1975

? 10000

MOST TAXPAYERS DON'T HAVE EXCLUDED  
FOREIGN INCOME, PENALIZED AMOUNTS  
UNDER CODE SEC 72(M)(5), OR EXCESS  
COMMUNITY INCOME. DO YOU HAVE ANY  
OF THESE ITEMS? (Y/N)

? N

ENTER TAXABLE INCOME FOR YEAR 1979  
? 52500

FOR URIAH HEEP, 1979 TAX,  
USING INCOME AVERAGING,  
COMES TO 11672.2

THE FOLLOWING REPRESENTS THE FILLED-IN  
SCHEDULE G, USING THE 1979 FORMAT:

\*\*\*\*\* SCHEDULE G \*\*\*\*\*

URIAH HEEP -- 1979

FILING STATUS: MARR./JOINT

ENTER 'C' TO CONTINUE? C

BASE PERIOD INCOME AND ADJUSTMENTS

LINE 1 -	1978 : \$ 16000
	1977 : \$ 14000
	1976 : \$ 0
	1975 : \$ 0
LINE 2 -	1978 : \$ 2250
	1977 : \$ 2250
	1976 : \$ 0
	1975 : \$ 0
LINE 3 -	1978 : \$ 13750
	1977 : \$ 11750
	1976 : \$ 12000
	1975 : \$ 10000
LINE 4 -	1978 : \$ 0
	1977 : \$ 0
	1976 : \$ 0
	1975 : \$ 0
LINE 5 -	1976 : \$ 3200
	1975 : \$ 3200
LINE 6 -	1978 : \$ 13750
	1977 : \$ 11750
	1976 : \$ 15200
	1975 : \$ 13200

ENTER 'C' TO CONTINUE? C

# COMPUTATION OF AVERAGEABLE INCOME

## AND COMPUTATION OF TAX

```

LINE 7 : $ 52500
LINE 8 : $ 0
LINE 9 : $ 52500
LINE 10 : $ 0
LINE 11 : $ 52500
LINE 12 : $ 53900
LINE 13 : $ 16170
LINE 14 : $ 36330
LINE 15 : $ 16170
LINE 16 : $ 7266
LINE 17 : $ 23436
ENTER 'C' TO CONTINUE? C
LINE 18 : $ 0
LINE 19 : $ 23436
LINE 20 : $ 4179.08
LINE 21 : $ 4179.08
LINE 22 : $ 2305.8
LINE 23 : $ 1873.28
LINE 24 : $ 7493.12
LINE 25 : $ 0
LINE 26 : $ 0
LINE 27 : $ 0
LINE 28 : $ 11672.2
***** END OF SCHEDULE G *****
ENTER 'C' TO CONTINUE WITH NEXT TAXPAYER? X

```

### Practice Problems

1. Hester Prynne is single, head of household, and has one dependent. Line 34 of her 1978 Form 1040 is \$13,988.39. Line 10 of her 1977 Form 1040A shows \$12,650.10. Her taxable income for 1976 was \$9,212.58; for 1975 it was \$8,775.34. In 1978, she had \$1,996.50 excluded under section 911. Her taxable income in 1979 is \$25,300.17, and she has \$1,100.00 subject to penalty under section 72(m)(5). How should she fill out her 1979 Schedule G?

Answer:

FOR HESTER PRYNNE, 1979 TAX,  
USING INCOME AVERAGING,  
COMES TO 5216.6

THE FOLLOWING REPRESENTS THE FILLED-IN  
SCHEDULE G, USING THE 1979 FORMAT:

```

***** SCHEDULE G *****
HESTER PRYNNE -- 1979
FILING STATUS: UNM. HEAD OF HOUSEHOLD

ENTER 'C' TO CONTINUE? C
BASE PERIOD INCOME AND ADJUSTMENTS
LINE 1 -          1978 : $ 13988.39
          1977 : $ 12650.1
          1976 : $ 0
          1975 : $ 0

```

LINE 2 -                   1978 : \$ 1500  
           1977 : \$ 1500  
           1976 : \$ 0  
           1975 : \$ 0  
 LINE 3 -                   1978 : \$ 12488.39  
           1977 : \$ 11150.1  
           1976 : \$ 9212.58  
           1975 : \$ 8775.34  
 LINE 4 -                   1978 : \$ 1996.5  
           1977 : \$ 0  
           1976 : \$ 0  
           1975 : \$ 0  
 LINE 5 -                   1976 : \$ 2200  
           1975 : \$ 2200  
 LINE 6 -                   1978 : \$ 14484.89  
           1977 : \$ 11150.1  
           1976 : \$ 11412.58  
           1975 : \$ 10975.34

ENTER 'C' TO CONTINUE? C

COMPUTATION OF AVERAGEABLE INCOME  
 AND COMPUTATION OF TAX

LINE 7 : \$ 25300.17  
 LINE 8 : \$ 1100  
 LINE 9 : \$ 24200.17  
 LINE 10 : \$ 0  
 LINE 11 : \$ 24200.17  
 LINE 12 : \$ 48022.91  
 LINE 13 : \$ 14406.87  
 LINE 14 : \$ 9793.3  
 LINE 15 : \$ 14406.87  
 LINE 16 : \$ 1958.66  
 LINE 17 : \$ 16365.53

ENTER 'C' TO CONTINUE? C

LINE 18 : \$ 0  
 LINE 19 : \$ 16365.53  
 LINE 20 : \$ 2831.04  
 LINE 21 : \$ 2831.04  
 LINE 22 : \$ 2333.65  
 LINE 23 : \$ 497.39  
 LINE 24 : \$ 1989.56  
 LINE 25 : \$ 5599.06  
 LINE 26 : \$ 5203.06  
 LINE 27 : \$ 396  
 LINE 28 : \$ 5216.6

\*\*\*\*\* END OF SCHEDULE G \*\*\*\*\*

ENTER 'C' TO CONTINUE WITH NEXT TAXPAYER? X

2. Billy Budd is single and has no dependents. Line 34 of his 1978 Form 1040 is \$45,130.75. Line 34 of his 1977 Form 1040 is \$48,968.20. In 1976 and 1975, his taxable incomes were \$37,500.00 and \$38,105.05. He had \$3,000.00 of excludable foreign income in 1978, \$3,000.00 in 1977, \$2,500.00 in 1976, and \$2,000.00 in 1975. He has no excess community income and nothing subject to section 72(m)(5) penalty. His income for 1979 is \$57,762.53. How would he complete Schedule G, if he is eligible for income averaging?

BILLY BUDD  
 DOES NOT QUALIFY FOR AVERAGING.  
 AVERAGEABLE INCOME FOR 1979  
 IS \$ 2831.33 - WHICH IS \$3000 OR LESS.  
 ENTER 'C' TO CONTINUE WITH NEXT TAXPAYER? X

### Program Listing

```

1 REM ///// INCOME AVERAGING /////
2 REM A() HOLDS SCHEDULE G AMOUNTS
3 REM C() AND R() ARE FOR TAX RATE SCHEDULES
10 DIM A(45),C(4,16),R(4,16)
19 REM READ TAX RATE SCHEDULES
20 GOSUB 6900
39 REM ROUNDOFF FUNCTION
40 DEF FNR(X)=INT(100*X+.5)/100
49 REM CLEAR SCHEDULE G FOR NEXT TAXPAYER
50 FOR I=1 TO 15
60 A(I)=0
70 NEXT I
79 REM CLEAR SCREEN
80 PRINT "3"
85 PRINT "INCOME AVERAGING"
90 PRINT "TAXPAYER'S NAME IS:"
100 INPUT Z$
105 PRINT
110 PRINT "TAXABLE YEAR:"
120 INPUT A(1)
125 PRINT
130 PRINT "ENTER FILING STATUS--"
140 PRINT " --1 FOR SINGLE"
150 PRINT " --2 FOR MARRIED/JOINT"
160 PRINT " --3 FOR MARRIED/SEPARATE"
170 PRINT " --4 FOR HEAD OF HOUSEHOLD"
180 PRINT " --5 FOR QUALIFYING WIDOW(ER)"
190 INPUT F
200 PRINT
256 REM
257 REM ***** BASE PERIOD INCOME AND ADJUSTMENTS *****

```

```

258 REM
268 REM ENTER INCOME AMOUNTS--
269 REM PROCEDURE IS DIFFERENT BEFORE 1977
270 FOR J=1 TO 4
280 IF A(1)-J>1976 THEN 320
290 PRINT "ENTER TAXABLE INCOME FOR YEAR";A(1)-J
300 INPUT A(J+9)
305 PRINT
310 GOTO 750
320 PRINT "ENTER THE INCOME FIGURE CORRESPONDING"
330 PRINT "   TO LINE 34 ON FORM 1040, OR ON FORM"
340 PRINT "   1040A, CORRESPONDING TO LINE 11(1979)"
350 PRINT "   OR LINE 10(1977-1978)....."
360 PRINT "FOR THE YEAR";A(1)-J
370 INPUT A(J+1)
380 PRINT
470 PRINT "HOW MANY EXEMPTIONS CLAIMED THAT YEAR?"
480 INPUT B
485 PRINT
488 REM EXEMPTIONS ARE $1000 EACH 1979 AND AFTER,
489 REM $750 EACH BEFORE THAT
490 A(J+5)=1000*B
500 IF A(1)-J>1978 THEN 740
510 A(J+5)=750*B
740 A(J+9)=A(J+1)-A(J+5)
750 NEXT J
866 REM 5. FROM FILING STATUS, DETERMINE ZERO
867 REM   BRACKET AMOUNT FOR 1975 AND 1976
868 REM IF TAX YEAR IS 1981 OR LATER, IGNORE
869 REM ZERO BRACKET AMOUNTS
870 IF A(1)>1980 THEN 1010
890 ON F GOTO 900,920,940,900,920
899 REM SINGLE OR HEAD OF HOUSEHOLD
900 A(15)=2200
910 GOTO 960
919 REM MARRIED/JOINT OR WIDOW(ER)
920 A(15)=3200
930 GOTO 960
939 REM MARRIED/SEPARATE
940 A(15)=1600
949 REM 1975 SAME AS 1976
958 REM IF TAX YEAR IS 1980, IGNORE 1975
959 REM ZERO BRACKET AMOUNT
960 IF A(1)=1980 THEN 1010
970 A(14)=A(15)
1010 PRINT "MOST TAXPAYERS DON'T HAVE EXCLUDED"
1020 PRINT "   FOREIGN INCOME, PENALIZED AMOUNTS"
1030 PRINT "   UNDER CODE SEC 72(M)(5), OR EXCESS"
1040 PRINT "   COMMUNITY INCOME. DO YOU HAVE ANY"
1050 PRINT "   OF THESE ITEMS? (Y/N)"
1060 INPUT W$
1070 IF W$="N" THEN 1200
1080 PRINT "EXCLUDED FOREIGN INCOME--YEAR";A(1)-1
1090 INPUT A(16)
1100 PRINT "                               SAME--YEAR";A(1)-2

```



```

1110 INPUT A(17)
1120 PRINT " SAME--YEAR";A(1)-3
1130 INPUT A(18)
1140 PRINT " SAME--YEAR";A(1)-4
1150 INPUT A(19)
1155 PRINT
1160 PRINT "ENTER PENALIZED AMOUNTS, SEC. 72(M)(5)"
1170 INPUT A(20)
1180 PRINT "ENTER EXCESS COMMUNITY INCOME"
1190 INPUT A(21)
1195 PRINT
1199 REM ADD UP BASE PERIOD INCOME COLUMNS A-D
1200 A(22)=A(10)+A(16)
1210 A(23)=A(11)+A(17)
1220 A(24)=A(12)+A(18)+A(14)
1230 A(25)=A(13)+A(19)+A(15)
1238 REM BASE PERIOD INCOME CANNOT BE NEGATIVE
1239 REM IN ANY YEAR
1240 FOR I=22 TO 25
1250 IF A(I)>0 THEN 1280
1270 A(I)=0
1280 NEXT I
1286 REM
1287 REM **** COMPUTATION OF AVERAGABLE INCOME ****
1288 REM
1289 REM 7. TAXABLE INCOME FROM SCHEDULE TC
1290 PRINT "ENTER TAXABLE INCOME FOR YEAR";A(1)
1300 INPUT A(26)
1305 PRINT
1309 REM 9. SUBTRACT LINE 8 FROM LINE 7
1310 A(27)=A(26)-A(20)
1318 REM 10. EXCESS COMMUNITY INCOME IS A(21)
1319 REM 11. ADJUSTED TAXABLE INCOME
1320 A(28)=A(27)-A(21)
1329 REM LINE 11 CANNOT BE NEGATIVE
1330 IF A(28)>=0 THEN 1360
1350 A(28)=0
1359 REM 12. TOTAL BASE PERIOD INCOME
1360 A(29)=A(22)+A(23)+A(24)+A(25)
1379 REM 13. 30% OF LINE 12
1380 A(30)=FNR(A(29)*.3)
1389 REM 14. AVERAGEABLE INCOME
1390 A(31)=A(28)-A(30)
1400 IF A(31)>=3000 THEN 1450
1420 PRINT Z$
1425 PRINT "DOES NOT QUALIFY FOR AVERAGING."
1430 PRINT "AVERAGEABLE INCOME FOR ";A(1)
1435 PRINT "IS $";A(31);"- WHICH IS $3000 OR LESS."
1440 GOTO 2170
1449 REM 15. AMOUNT FROM LINE 13
1450 A(32)=A(30)
1469 REM 16. 20% OF LINE 14
1470 A(33)=FNR(A(31)*.2)
1479 REM 17. TOTAL (ADD LINES 15 AND 16)
1480 A(34)=A(32)+A(33)
1488 REM 18. EXCESS COMMUNITY INCOME IS A(21)

```

```

1489 REM 19. TOTAL (ADD LINES 17 AND 18)
1490 A(35)=A(34)+A(21)
1499 REM 20. TAX ON LINE 19 AMOUNT
1500 S=A(35)
1510 GOSUB 6000
1520 A(36)=T
1529 REM 21. TAX ON LINE 17 AMOUNT
1530 S=A(34)
1540 GOSUB 6000
1550 A(37)=T
1559 REM 22. TAX ON LINE 15 AMOUNT
1560 S=A(32)
1570 GOSUB 6000
1580 A(38)=T
1589 REM 23. SUBTRACT LINE 22 FROM LINE 21
1590 A(39)=A(37)-A(38)
1599 REM 24. MULTIPLY LINE 23 AMOUNT BY 4
1600 A(40)=4*A(39)
1608 REM -IF THERE'S NO SECTION 72(M)(5) PENALTY
1609 REM -INCOME, SKIP TO LINE 28
1610 IF A(20)=0 THEN 1690
1619 REM 25. TAX ON LINE 7 AMOUNT
1620 S=A(26)
1630 GOSUB 6000
1640 A(41)=T
1649 REM 26. TAX ON LINE 9 AMOUNT
1650 S=A(27)
1660 GOSUB 6000
1670 A(42)=T
1679 REM 27. SUBTRACT LINE 26 FROM LINE 25
1680 A(43)=A(41)-A(42)
1689 REM 28. TAX (ADD LINES 20, 24, AND 27)
1690 A(44)=A(36)+A(40)+A(43)
1692 REM
1693 REM ***** PRINT SCHEDULE G *****
1694 REM
1695 PRINT "FOR ";Z$;" ";A(1);"TAX,"
1700 PRINT "USING INCOME AVERAGING,"
1710 PRINT "COMES TO";A(44)
1720 PRINT
1730 PRINT "THE FOLLOWING REPRESENTS THE FILLED-IN"
1740 PRINT "SCHEDULE G, USING THE 1979 FORMAT:"
1750 PRINT
1755 PRINT "***** SCHEDULE G *****"
1760 PRINT Z$;" --";A(1)
1770 PRINT "FILING STATUS: ";
1780 ON F GOTO 1790,1810,1830,1850,1870
1790 PRINT "SINGLE"
1800 GOTO 1880
1810 PRINT "MARR./JOINT"
1820 GOTO 1880
1830 PRINT "MARR./SEP."
1840 GOTO 1880
1850 PRINT "UNM. HEAD OF HOUSEHOLD"
1860 GOTO 1880

```

```

1870 PRINT "QUAL. WIDOW(ER)"
1880 PRINT
1889 REM WAIT FOR OPERATOR CUE TO CONTINUE
1890 GOSUB 5800
1895 PRINT "BASE PERIOD INCOME AND ADJUSTMENTS"
1899 REM PRINT LINES 1, 2, AND 3
1900 FOR I=2 TO 10 STEP 4
1910 GOSUB 5700
1915 NEXT I
1919 REM PRINT LINE 4
1920 I=16
1930 GOSUB 5700
1939 REM PRINT LINE 5, IF IT'S APPLICABLE
1940 PRINT "LINE 5 - ";
1950 IF A(14)=0 THEN 1970
1960 PRINT ,A(1)-3;": $";A(14)
1970 IF A(15)=0 THEN 1990
1980 PRINT ,A(1)-4;": $";A(15)
1989 REM PRINT LINE 6
1990 I=22
2000 GOSUB 5700
2005 PRINT
2009 REM WAIT FOR OPERATOR CUE TO CONTINUE
2010 GOSUB 5800
2015 PRINT
2020 PRINT "COMPUTATION OF AVERAGEABLE INCOME"
2030 PRINT "      AND COMPUTATION OF TAX"
2040 PRINT "LINE 7 : $";A(26)
2050 PRINT "LINE 8 : $";A(20)
2060 PRINT "LINE 9 : $";A(27)
2070 PRINT "LINE 10 : $";A(21)
2080 FOR J=11 TO 17
2090 PRINT "LINE";J;": $";A(J+17)
2100 NEXT J
2109 REM WAIT FOR OPERATOR CUE TO CONTINUE
2110 GOSUB 5800
2120 PRINT "LINE 18 : $";A(21)
2130 FOR J=19 TO 28
2140 PRINT "LINE";J;": $";A(J+16)
2150 NEXT J
2160 PRINT "***** END OF SCHEDULE G *****"
2168 REM WAIT BEFORE ERASING SCREEN FOR
2169 REM NEXT TAXPAYER
2170 PRINT "ENTER 'C' TO CONTINUE WITH NEXT TAXPAYER";
2180 INPUT W$
2190 IF W$="C" THEN 50
3000 END
5697 REM
5698 REM *** SUBROUTINE TO PRINT ALL OF LINE 1,2,3,4,OR 6 ***
5699 REM
5700 PRINT "LINE";INT((I-2)/4)+1;"- ";
5710 FOR J=0 TO 3
5720 PRINT ,A(1)-J-1;": $";A(I+J)
5730 NEXT J
5740 RETURN

```

```

5795 REM
5796 REM *** SUBROUTINE TO WAIT FOR OPERATOR CUE
5797 REM      TO CONTINUE SINCE ENTIRE SCHEDULE G
5798 REM      WON'T FIT ON ONE SCREEN ***
5799 REM
5800 PRINT "ENTER 'C' TO CONTINUE";
5810 INPUT W$
5820 RETURN
5994 REM
5995 REM *** SUBROUTINE TO CALCULATE TAX ON AMOUNT S ***
5996 REM
5999 REM INITIALIZE TAX TO ZERO
6000 T=0
6002 REM SINGLE HAS 16 BRACKETS, ALL OTHERS HAVE 15
6003 K=15
6004 IF F>1 THEN 6010
6005 K=16
6009 REM DETERMINE WHETHER TO USE SCHED. X,Y, OR Z
6010 I=F
6019 REM WIDOW(ER) SAME AS MARRIED/JOINT
6020 IF F<5 THEN 6040
6030 I=2
6039 REM START WITH ZERO BRACKET AMOUNT
6040 J=1
6049 REM IS INCOME <= ZERO BRACKET AMOUNT?
6050 IF S<=C(I,J) THEN 6130
6059 REM IS INCOME > THIS BRACKET'S CEILING?
6060 IF S>C(I,J+1) THEN 6090
6068 REM FOUND MAX TAX BRACKET--
6069 REM --TAX BALANCE OF INCOME
6070 T=T+(S-C(I,J))*R(I,J)/100
6080 GOTO 6130
6089 REM ACCUMULATE TAX FROM THIS BRACKET
6090 T=T+(C(I,J+1)-C(I,J))*R(I,J)/100
6099 REM PROCEED TO NEXT BRACKET
6100 J=J+1
6110 IF J<K THEN 6060
6119 REM TAX BALANCE OF INCOME AT HIGHEST RATE
6120 T=T+(S-C(I,J))*R(I,J)/100
6129 REM ROUND TAX AMOUNT
6130 T=FNR(T)
6140 RETURN
6897 REM
6898 REM *** SUBROUTINE TO READ TAX RATES ***
6899 REM
6900 RESTORE
6909 REM FIRST SCHED X
6910 FOR J=1 TO 16
6920 READ R(1,J),C(1,J)
6930 NEXT J
6939 REM THEN SCHEDS Y & Z
6940 FOR I=2 TO 4
6950 FOR J=1 TO 15
6960 READ R(I,J),C(I,J)
6970 NEXT J

```

```

6980 NEXT I
6985 RETURN
6990 REM
6991 REM ***** 1979 TAX RATE SCHEDULES X,Y, AND Z *****
6992 REM
6993 REM FOR EACH TABLE BELOW, GET RATE AND
6994 REM CUTOFF DATA PAIR FROM THE RIGHTMOST TWO
6995 REM COLUMNS OF THE APPROPRIATE SCHEDULE
6996 REM
6997 REM ----SCHEDULE X----
6998 REM
7000 DATA 14,2300,16,3400,18,4400,19,6500,21,8500
7005 DATA 24,10800,26,12900,30,15000,34,18200
7010 DATA 39,23500,44,28800,49,34100,55,41500
7020 DATA 63,55300,68,81800,70,108300
7027 REM
7028 REM ----SCHEDULE Y (JOINT/WIDOW)----
7029 REM
7030 DATA 14,3400,16,5500,18,7600,21,11900,24,16000,28
7040 DATA 20200,32,24600,37,29900,43,35200,49,45800,54
7050 DATA 60000,59,85600,64,109400,68,162400,70,215400
7057 REM
7058 REM ----SCHEDULE Y (SEPARATE)----
7059 REM
7060 DATA 14,1700,16,2750,18,3800,21,5950,24,8000,28,10100
7070 DATA 32,12300,37,14950,43,17600,49,22900,54,30000
7080 DATA 59,42800,64,54700,68,81200,70,107700
7087 REM
7088 REM ----SCHEDULE Z----
7089 REM
7090 DATA 14,2300,16,4400,18,6500,22,8700,24,11800,26,15000
7100 DATA 31,18200,36,23500,42,28800,46,34100,54,44700,59
7110 DATA 60600,63,81800,68,108300,70,161300
9999 END

```

## References

- U.S. Internal Revenue Service Code, Sections 1301-1305.
- U.S. Public Law 91-172, Section 311(b) amending Internal Revenue Code Section 1302.
- U.S. Treasury Department, Internal Revenue Service. *Income Averaging*, publication number 506.
- U.S. Treasury Department, Internal Revenue Service. Regulations, Sections 1.1301-0 to 1304-6, especially the last sentence of 1.1302-02(b)(1).

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## Current Value of a Treasury Bill

---

Treasury bills differ from other investment vehicles in that they are bought and sold at a discount from their face value. The rate will vary as the bill approaches maturity. Also, discounts are figured as if a year were 360 days; the annual percentage rate, or yield, is calculated using a 365/366-day year.

To use this program, enter the T-bill's face value, issue and maturity dates in MONTH, DAY, YEAR format, using one or two numbers for each value (be sure to separate each value with a comma). Then enter the current date and current price bid. The program provides the current value as a dollar amount.

### Example

A \$10,000 T-bill was sold 1/10/80 to mature on 4/10/80. On 1/17/80, government securities dealers were quoting a bid price of 12.09%. How much was the bill worth?

Answer: The bill was worth \$9,717.90

### CURRENT VALUE OF A TREASURY BILL

```
                FACE VALUE ($) ? 10000
            ISSUE DATE (MM,DD,YY) ? 1,10,80
        MATURITY DATE (MM,DD,YY) ? 4,10,80
        TODAY'S DATE (MM,DD,YY) ? 1,17,80
        CURRENT PRICE BID (%) ? 12.09
```

CURRENT VALUE = \$ 9717.9

WOULD YOU LIKE TO RE-RUN THIS PROGRAM USING NEW DATA (Y/N) ? N

### Practice Problems

1. A one year bill issued 2/16/80 with a face value of \$50,000 was sold on 4/10/80 at a 7.35% discount. What was the selling price?

Answer: The bill sold for \$46,815.00.

2. Diego bought a \$1,000,000.00 bill on 1/25/80 that matures 7/25/80. On 4/10/80 he noted that dealers were offering 15.54% on his issue. How much could Diego sell his bill for on that day?

Answer: The bill was worth \$954,243.33.

### Program Listing

```
10 PRINT "CURRENT VALUE OF A TREASURY BILL"
20 DEF FNA(X)=INT(X*100+.5)/100
30 PRINT
40 PRINT "                FACE VALUE ($) ";
50 INPUT P
60 PRINT "                ISSUE DATE (MM,DD,YY) ";
70 INPUT M,D,Y
80 GOSUB 340
90 REM -- X3 = ABSOLUTE NUMBER OF DAYS FROM IMAGINARY DATE
```

```

100 REM --      00/00/00 TO ISSUE DATE
110 X3=A4
120 PRINT "      MATURITY DATE (MM,DD,YY)";
130 INPUT M,D,Y
140 GOSUB 340
150 REM -- X4 = TOTAL NUMBER OF DAYS IN PERIOD
160 X4=ABS(X3-A4)
170 PRINT "      TODAY'S DATE (MM,DD,YY)";
180 INPUT M,D,Y
190 GOSUB 340
200 REM -- X3 = NUMBER OF DAYS FROM ISSUE TO TODAY
210 X3=ABS(X3-A4)
220 PRINT "      CURRENT PRICE BID (%)" ;
230 INPUT B
240 REM -- X4 = NUMBER OF DAYS LEFT UNTIL MATURITY
250 X4=X4-X3
260 PRINT
270 PRINT "CURRENT VALUE = $";FNA(P-((P/1E4)*(B*(X4/360)*100)))
280 PRINT
290 PRINT "WOULD YOU LIKE TO RE-RUN THIS PROGRAM USING NEW DATA
      (Y/N)";
300 INPUT Z$
310 IF Z$="Y" THEN 30
320 IF Z$="N" THEN 450
330 GOTO 290
340 REM -- SUBROUTINE TO DETERMINE NUMBER OF DAYS BETWEEN IMAGINARY
350 REM -- DATE 00/00/00 AND MM/DD/YY USING 365/366 DAY YEAR.
360 REM -- REF. ACCOUNTS PAYABLE & ACCOUNTS RECEIVABLE (WANG), P.
      255
370 RESTORE
380 DATA 0,3,3,6,8,11,13,16,19,21,24,26
390 FOR I1=1 TO M
400 READ A4
410 NEXT I1
420 A4=A4+Y*365+INT(Y/4)+1+(M-1)*28+D
430 IF INT(Y/4)=Y/4 AND M<3 THEN A4=A4-1
440 RETURN
450 END

```

## References

- U.S. Department of Treasury. *Information about Treasury Bills Sold at Original Issue*, Form PD 800-D (rev. June 1978).
- U.S. Federal Reserve. *Marketable Securities of the United States Government — U.S. Treasury Bills, Notes, and Bonds*, circular No. LLM 185.



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## Accrued Interest on Bonds

---

This program computes accrued interest to date on a bond. The program performs calculation using either a 365/366-day standard year, or a 360-day year method (used by some federal agency notes and bonds). Sometimes a bond is issued after the first period has begun. Because this results in a first coupon payment of less than the normal amount, some issues skip that payment and include it with the second period's payment. In this case, you would respond "Y" for Yes when the program asks if this coupon involves a long first period, and enter the additional dates requested.

To use the program, select the type of year the bond calculations will use, then enter the coupon rate and the number of coupons per year. If this coupon involves a long first coupon, enter a "Y" and enter the date the first period began, the date the bond was acquired, and the date the first coupon would normally have been paid had this not been a long coupon. If this coupon is normal or short, enter "N" and then enter the beginning date for this period. For both long and normal or short coupons, you now enter the date the current period ends, and the settlement date. The program will output the accrued interest in percent of par value.

### Example

What is the accrued interest for settlement on 9/10/79, for an 8.25% note due 8/31/81 and issued on 8/29/79, with a long first coupon? The coupon dates are 2/28 and 8/31. The first period began on 2/28/79. (1980 is a leap year, so the end of the current period is 2/29/80.)

Answer: Accrued interest is .271485% of par value.

ACCRUED INTEREST ON BONDS

COMPUTE USING:

- 1) 360 DAY YEAR
- 2) 365/366 DAY YEAR
- 3) END PROGRAM

WHICH? 2

COUPON RATE (%)? 8.25

NUMBER OF COUPONS PER YEAR? 2

DOES THIS COUPON INCLUDE A LONG FIRST PERIOD (Y/N)? Y

BEGINNING OF FIRST PERIOD (MM,DD,YY)? 2,28,79

ISSUE DATE (MM,DD,YY)? 8,29,79

FIRST COUPON DATE (MM,DD,YY)? 8,31,79

END OF CURRENT PERIOD (MM,DD,YY)? 2,29,80

SETTLEMENT DATE (MM,DD,YY)? 9,10,79

ACCRUED INTEREST IS .271485 % OF PAR.

WOULD YOU LIKE TO RE-RUN THIS PROGRAM USING NEW DATA (Y/N)? N

## Practice Problem

What is the accrued interest for settlement on 6/3/80, of a Federal Home Loan Bank Bond at 7.375% due 8/25/82? The coupon payment dates are 2/25 and 8/25. (FHLB bonds use a 360-day year for calculations.)

Answer: 2.007638% of par.

## Program Listing

```
10 PRINT "ACCRUED INTEREST ON BONDS"
20 PRINT
30 PRINT "COMPUTE USING:"
40 PRINT "                1) 360 DAY YEAR"
50 PRINT "                2) 365/366 DAY YEAR"
60 PRINT "                3) END PROGRAM"
70 PRINT
80 PRINT "                WHICH";
90 INPUT T
100 IF T=1 THEN 130
110 IF T=3 THEN 820
120 IF T<>2 THEN 80
130 PRINT
140 PRINT "                COUPON RATE (%)"
150 INPUT I
160 PRINT
170 PRINT " NUMBER OF COUPONS PER YEAR";
180 INPUT N
190 X1=0
200 PRINT
210 PRINT "DOES THIS COUPON INCLUDE A LONG FIRST PERIOD (Y/N)";
220 INPUT Z$
230 IF Z$="N" THEN 410
240 IF Z$<>"Y" THEN 210
250 REM -- SKIP THIS SECTION IF FIRST PERIOD IS NOT LONG
260 PRINT
270 PRINT " BEGINNING OF FIRST PERIOD (MM,DD,YY)";
280 GOSUB 650
290 X2=A4
300 REM -- ISSUE DATE IS DATE CURRENT BONDHOLDER OBTAINED THE BOND
310 PRINT "                ISSUE DATE (MM,DD,YY)";
320 GOSUB 650
330 REM -- X1 = NUMBER OF DAYS FROM ISSUE TO END OF PARTIAL PERIOD
340 X1=ABS(X2-A4)
350 PRINT "                FIRST COUPON DATE (MM,DD,YY)";
360 GOSUB 650
370 REM -- X2 = TOTAL NUMBER OF DAYS IN FIRST PERIOD
380 X2=ABS(X2-A4)
390 X1=(X2-X1)/X2
400 GOTO 460
410 PRINT
420 PRINT "BEGINNING OF CURRENT PERIOD (MM,DD,YY)";
430 GOSUB 650
440 REM -- X3 = ABSOLUTE NUMBER OF DAYS FROM IMAGINARY DATE
450 REM --      00/00/00 TO BEGINNING OF CURRENT PERIOD
460 X3=A4
```

```

470 PRINT "          END OF CURRENT PERIOD (MM,DD,YY)";
480 GOSUB 650
490 REM -- X4 = TOTAL NUMBER OF DAYS IN CURRENT PERIOD
500 X4=ABS(X3-A4)
510 PRINT "          SETTLEMENT DATE (MM,DD,YY)";
520 GOSUB 650
530 REM -- X3 = NUMBER OF DAYS FROM BEGINNING OF
540 REM --      CURRENT PERIOD TO SETTLEMENT DATE
550 X3=ABS(X3-A4)
560 X3=(X3/X4)+X1
570 PRINT
580 PRINT "ACCRUED INTEREST IS";(I/N)*X3;"% OF PAR."
590 PRINT
600 PRINT "WOULD YOU LIKE TO RE-RUN PROGRAM USING NEW DATA (Y/N)";
610 INPUT Z$
620 IF Z$="Y" THEN 20
630 IF Z$="N" THEN 820
640 GOTO 600
650 INPUT M,D,Y
660 IF T=1 THEN 800
670 REM -- SUBROUTINE TO DETERMINE NUMBER OF DAYS BETWEEN IMAGINA
680 REM -- RY DATE 00/00/00 AND MM/DD/YY USING 365/366 DAY YEAR.
690 REM -- REF. ACCOUNTS PAYABLE & ACCOUNTS RECEIVABLE (WANG), P.
    255
700 RESTORE
710 DATA 0,3,3,6,8,11,13,16,19,21,24,26
720 FOR I1=1 TO M
730 READ A4
740 NEXT I1
750 A4=A4+Y*365+INT(Y/4)+1+(M-1)*28+D
760 IF INT(Y/4)<>Y/4 THEN 770
762 IF M>2 THEN 770
764 A4=A4-1
770 RETURN
780 REM -- SUBROUTINE TO COMPUTE NUMBER OF DAYS FROM
790 REM -- IMAGINARY DATE 00/00/00 TO MM/DD/YY USING 360 DAY YEAR.
800 A4=(Y*360)+(M*30)+D
810 RETURN
820 END

```

## Reference

Stigum, Marcia. *The Money Market: Myth, Reality, and Practice*. Homewood, Ill.: Dow Jones-Irwin, 1978.  
 Pages 538-47.

---

## Continuous Interest Compounding

---

This program calculates the future value of an investment for which interest is compounded continuously. You must enter the interest rate, the number of years that interest will accrue, and the amount of the initial deposit. The total value is based on the following formula:

$$T = De^{IN}$$

where:

T = total value after N years

D = initial investment

I = interest rate

e = 2.718281828... (base of natural logarithms)

### Example

Dan deposits \$800.00 at 7½% interest, compounded continuously. How much will his account be worth in ten years?

Answer: \$1,693.60

```
CONTINUOUS INTEREST COMPOUNDING
ENTER THE ANNUAL INTEREST RATE
TO BE PAID ON THE ACCOUNT
? 7.5
ENTER THE NUMBER OF YEARS OR FRACTIONS
OF YEARS THAT INTEREST WILL ACCRUE
? 10
ENTER YOUR INITIAL DEPOSIT
? 800
WITH CONTINUOUS COMPOUNDING A DEPOSIT OF
$ 800 GROWS IN 10 YEARS AT 7.5 % TO
$ 1693.6
```

### Practice Problems

1. If George invests \$5,000.00 at 9%, compounded continuously, how much will he have in 7 years and 3 months? (Enter 7 years 3 months as 7.25 years.)

Answer: \$9,601.68

2. Dr. Williams invests \$70.00 for his niece on the day she is born. How much will she get when she turns 21, at 6¼% compounded continuously?

Answer: \$260.08

## Program Listing

```
10 PRINT "CONTINUOUS INTEREST COMPOUNDING"
20 PRINT "ENTER THE ANNUAL INTEREST RATE"
30 PRINT "TO BE PAID ON THE ACCOUNT"
40 INPUT I
50 IF I<=0 THEN 20
60 PRINT "ENTER THE NUMBER OF YEARS OR FRACTIONS"
70 PRINT "OF YEARS THAT INTEREST WILL ACCRUE"
80 INPUT N
90 IF N<=0 THEN 60
100 PRINT "ENTER YOUR INITIAL DEPOSIT"
110 INPUT D
120 IF D<=0 THEN 100
130 PRINT "WITH CONTINUOUS COMPOUNDING A DEPOSIT OF"
140 PRINT "$";D;" GROWS IN";N;" YEARS AT";I;"% TO"
150 PRINT "$";INT(100*(D*EXP(I/100*N))+.5)/100
160 END
```

---

## Rule of 78's Interest

---

This program computes the interest for each month of a loan in accordance with the rule of 78's. You enter the total interest which would have been earned had the loan continued to maturity, and the number of months in the original period of the loan. The program then prints out a table, with the number of each month, the interest earned during that month by the rule, the interest earned so far, and the balance of (unearned) interest remaining at the end of that month.

### Example

A 24-month loan calls for total interest of \$10,000.00. What is the interest for each month of the loan?

Answer:

```
RULE OF 78'S INTEREST
ENTER TOTAL INTEREST TO BE EARNED
TO MATURITY OF THE LOAN
? 10000
ENTER NO. OF MONTHS DURATION
OF THE LOAN TO MATURITY
? 24
MONTH      MONTH'S  ACCUM.  BAL. OF
OF LOAN    INTEREST  INT.    INTEREST
1          800      800     9200
2          766.67  1566.67 8433.33
3          733.33  2300     7700
4          700      3000     7000
5          666.67  3666.67 6333.33
6          633.33  4300     5700
7          600      4900     5100
8          566.67  5466.67 4533.33
9          533.33  6000     4000
10         500      6500     3500
11         466.67  6966.67 3033.33
12         433.33  7400     2600
13         400      7800     2200
14         366.67  8166.67 1833.33
15         333.33  8500     1500
16         300      8800     1200
17         266.67  9066.67 933.33
18         233.33  9300     700
19         200      9500     500
20         166.67  9666.67 333.33
21         133.33  9800     200
22         100      9900     100
23         66.67   9966.67 33.33
24         33.33   10000    0
PENNY BREAKAGE ADJUSTED IN LAST MONTH
```

### Practice Problems

1. Laurie took out a 36-month loan. Her total interest was \$3,614.59. What was the balance of unearned interest if she terminated the loan after 2 years?

Answer: \$423.33

2. Bob Johnson pays off a 3-year loan 2 years early. If the total interest would have been \$180.00, how much interest did he actually pay?

Answer: \$98.94

### Program Listing

```
5 PRINT "RULE OF 78'S INTEREST"
9 REM ROUND OFF FUNCTION
10 DEF FNR(X)=INT(100*X+.5)/100
20 PRINT "ENTER TOTAL INTEREST TO BE EARNED"
30 PRINT "TO MATURITY OF THE LOAN"
40 INPUT I
60 PRINT "ENTER NO. OF MONTHS DURATION"
70 PRINT "OF THE LOAN TO MATURITY"
80 INPUT T
100 T1=T*(T+1)/2
110 PRINT "MONTH", "MONTH'S", "ACCUM.", "BAL. OF"
120 PRINT "OF LOAN", "INTEREST", "INT.", "INTEREST"
130 A=0
139 REM PRINT TABLE
140 FOR M=1 TO T-1
170 J=FNR((T-M+1)*I/T1)
180 A=A+J
190 B=I-A
240 PRINT M, J, A, FNR(B)
250 NEXT M
255 PRINT T, FNR(B), A+B, 0
260 PRINT "PENNY BREAKAGE ADJUSTED IN LAST MONTH"
270 END
```



---

## Present Value of a Tax Deduction

---

When evaluating an investment, the value of the tax savings is often a consideration. This program calculates the amount of any savings you might realize by deducting interest payments.

You must enter the tax rate, the interest rate on the debt, the term of the debt (in years), and the amount of interest to be paid during each year of the term.

### Program Notes

If the level of debt will be constant throughout the term of the investment, you may want to change the program to calculate interest amounts as a percentage of a fixed dollar debt amount. Make these changes.

```
90 PRINT "NUMBER OF PERIODS";
100 INPUT N
102 PRINT "ENTER THE AMOUNT OF DEBT ($)";
104 INPUT Z
110 P=0
120 FOR J=1 TO N
    .
    .
    .
180 PRINT Z*K
190 P=P+(Z*K*T)/((1+K)^J)
200 NEXT J
```

### Example

What is the present value of the tax savings on projected interest payments of \$4,000, \$3,500, \$4,500, \$4,000, and \$5,000 over the next 5 years if the tax rate is 48% and the interest rate on that debt will be 19%?

Answer: If the 5 interest payments are deducted from taxable income, the present value of the taxes saved is \$6,044.74.

PRESENT VALUE OF AN INTEREST TAX DEDUCTION

```
WHAT IS THE TAX RATE (%)? 48
ENTER THE INTEREST RATE (%)? 19
NUMBER OF PERIODS? 5
INTEREST AMOUNT FOR PERIOD ($) 1 ? 4000
                                2 ? 3500
                                3 ? 4500
                                4 ? 4000
                                5 ? 5000
```

PRESENT VALUE OF DEDUCTION = \$ 6044.74

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA (Y/N)? N

## Practice Problems

1. If Nick buys a new truck for the shipping business he plans to start, the principal will be \$6,250.00 and the interest rate 16%. Nick will make interest payments of \$1,000.00, \$900.00, and \$800.00 during the 3 year term of the loan. If his new company will be in a 33% tax bracket, what is the present value of the taxes he will not have to pay when he deducts the interest payments?

Answer: The present value of the tax savings realized by deducting the interest payments is \$674.34.

2. If the tax rate is 30% and the interest rate is 15%, what is the present value of taxes saved by deducting interest payments of \$45.00, \$40.00, \$35.00, and \$30.00 during the next 4 years?

Answer: The present value of the tax savings here is \$32.86.

## Program Listing

```
10 PRINT "PRESENT VALUE OF AN INTEREST TAX DEDUCTION"
20 PRINT
30 PRINT "WHAT IS THE TAX RATE (%)"
40 INPUT T
50 T=T/100
60 PRINT "ENTER INTEREST RATE (%)"
70 INPUT K
80 K=K/100
90 PRINT "NUMBER OF PERIODS"
100 INPUT N
110 P=0
120 FOR J=1 TO N
130 IF J>1 THEN 160
140 PRINT "INTEREST AMOUNT FOR PERIOD ($)";
150 GOTO 170
160 PRINT " ";
170 PRINT J;
180 INPUT Z
190 P=P+(Z*T)/((1+K)^J)
200 NEXT J
210 PRINT
220 PRINT "PRESENT VALUE OF DEDUCTION = $";INT(P*100+.5)/100
230 PRINT
240 PRINT "WOULD YOU LIKE TO RE-RUN PROGRAM WITH NEW DATA (Y/N)";
250 INPUT Z$
260 IF Z$="Y" THEN 20
270 IF Z$<>"N" THEN 240
280 END
```

## References

Solomon and Pringle. *An Introduction to Financial Management*. Santa Monica, Calif.: Goodyear Publishing Co., 1977. Pages 376-78.

---

## Future Value of an Investment (Uneven Cash Flow)

---

Often it is useful to project the future (or terminal) value of monies to be received from an investment. The accept/reject criterion stipulates you should reject any investment whose future value of all cash flows, including the initial investment, is less than zero. This program computes that value, based on the term (in years), the growth rate, and the cash flow amounts for each year. The growth rate should be the rate at which you have alternative opportunities to invest.

### Program Notes

If you wish to compute the future value of cash flows over more than 10 years, you must change the size of array C( ) where it is dimensioned in line 30.

### Example

Aunt Lonna wants to start a college fund for her nephew Brian. She plans to put \$200.00 into savings this year, \$350.00 next year, and \$250.00 the following year. The interest rate is 6%. What will Brian's fund be worth at the end of the third year?

Answer: Brian's fund will be worth \$845.72.

```
FUTURE VALUE OF AN INVESTMENT
```

```
NUMBER OF CASH FLOWS? 3
```

```
GROWTH RATE (%)? 6
```

```
(ENTER INFLOWS AS POSITIVE VALUES, OUTFLOWS AS NEGATIVE)
```

```
AMOUNT OF CASH FLOW 1 ? 200
```

```
2 ? 350
```

```
3 ? 250
```

```
FUTURE VALUE AT END OF PERIOD 3 = $ 845.72
```

```
DO YOU WANT TO RE-RUN THIS PROGRAM WITH NEW DATA? (Y/N)? N
```

### Practice Problems

1. What will the value of \$25,000 be in 8 years if another \$25,000 is invested in year three and \$10,000 is withdrawn during the fifth year? The growth rate is 15%.

Answer: \$101,575.68.

2. If the growth rate above was 18%, what would the future value be?

Answer: \$120,400.47.

## Program Listing

```
10 PRINT "FUTURE VALUE OF AN INVESTMENT"
20 DEF FNA(X)=INT(X*100+.5)/100
30 PRINT
40 PRINT "  NUMBER OF CASH FLOWS";
50 INPUT N
60 PRINT "          GROWTH RATE (%)"
70 INPUT R
80 R=R/100
90 PRINT
100 T=0
110 PRINT "(ENTER INFLOWS AS POSITIVE , OUTFLOWS AS NEGATIVE)"
120 FOR J=1 TO N
130 IF J>1 THEN 160
140 PRINT "AMOUNT OF CASH FLOW";
150 GOTO 170
160 PRINT "          ";
170 PRINT J;
180 INPUT C(J)
190 REM ADD FUTURE VALUES OF EACH YEAR BASED ON RATE OF R
200 T=T+FNA(C(J)*(1+R)^(N-J))
210 NEXT J
220 PRINT
230 PRINT "          FUTURE VALUE AT END OF PERIOD";N;"= $";T
240 REM RESTART OR END PROGRAM?
250 PRINT
260 PRINT "DO YOU WANT TO RE-RUN PROGRAM WITH NEW DATA? (Y/N)";
270 INPUT Z$
280 IF Z$="Y" THEN 30
290 IF Z$<>"N" THEN 260
300 END
```

## Reference

Solomon and Pringle. *An Introduction to Financial Management*. Santa Monica, Calif.: Goodyear Publishing, 1977.

---

## Net Present Value of an Investment

---

Net Present Value (NPV) is defined as the present value of all cash flows associated with an investment, including the initial outlay. The NPV accept/reject criterion for an investment is: accept any investment whose NPV is greater than zero.

To use this program, you first enter the term of the investment (in years), the amount of the initial outlay, cash flow amounts for each year, and the required rate of return.

### Program Notes

To obtain the present value of an investment, enter an initial investment of zero.

### Example

Jack has an investment opportunity that requires an initial investment of \$10,000 and offers cash returns of \$3,000, \$5,000, and \$4,000 over the next 3 years. Jack wants at least 15% return on his money. What is the NPV of this investment? Should Jack accept?

Answer: The NPV of this investment is -\$980.52. Jack should not accept.

NET PRESENT VALUE

INVESTMENT? 10000

NUMBER OF YEARS? 3

REQUIRED RATE OF RETURN (%)? 15

ENTER CASH FLOW AMOUNTS FOR EACH YEAR (ENTER OUTFLOWS AS NEGATIVE).

INFLOW FOR YEAR 1 ? 3000

2 ? 5000

3 ? 4000

NET PRESENT VALUE = \$-980.52

DO YOU WANT TO RE-RUN THE PROGRAM WITH NEW DATA? (Y/N)? N

### Practice Problems

1. Doris holds a note for \$1,000.00 which matures in 2 years, but she wants to invest that money now in new sound equipment. Her bank will buy the note at a 10% discount. What price is the bank offering? (Hint: This is a *present* value calculation.)

Answer: The bank will pay Doris \$826.45 for the note.

2. What is the NPV of a \$1,500 investment which offers returns of \$800.00 year 1, \$900.00 year 2, requires \$1,000 more to be invested year 3, returns \$900.00 year 4, and \$800.00 year 5? Comparable 5 year investments currently offer a 15% return.

Answer: The NPV of this investment is \$130.98, quite acceptable.

## Program Listing

```
10 PRINT "NET PRESENT VALUE"
20 DEF FNA(X)=INT(X*100+.5)/100
30 REM CHANGE SIZE OF ARRAY C() TO MAXIMUM NUMBER OF CASH FLOWS
40 DIM C(10)
50 PRINT
60 PRINT "INVESTMENT";
70 INPUT C0
80 C0=-FNA(C0)
90 PRINT "NUMBER OF YEARS";
100 INPUT N
110 PRINT "REQUIRED RATE OF RETURN (%)";
120 INPUT R
130 R=R/100+1
140 F=0
150 PRINT
160 PRINT "ENTER CASH FLOW AMOUNTS EACH YEAR (ENTER OUTFLOWS AS
    NEGATIVE).";
170 PRINT
180 FOR J=1 TO N
190 IF J>1 THEN 220
200 PRINT "INFLOW FOR YEAR";
210 GOTO 230
220 PRINT "          ";
230 PRINT J;
240 INPUT C(J)
250 F=F+FNA(C(J))
260 NEXT J
270 T=C0
280 REM ADD PRESENT VALUES FOR EACH YEAR BASED ON RATE OF R
290 FOR J=1 TO N
300 T=T+FNA(C(J)/(R^J))
310 NEXT J
320 PRINT
330 PRINT "NET PRESENT VALUE = $";T
340 REM RESTART OR END PROGRAM?
350 PRINT
360 PRINT "DO YOU WANT TO RE-RUN PROGRAM WITH NEW DATA? (Y/N)";
370 INPUT Z$
380 IF Z$="Y" THEN 50
390 IF Z$<>"N" THEN 360
400 END
```

## References

- Rosen, Lawrence R. *Dow Jones-Irwin Guide to Interest*. Homewood, Ill.: Dow Jones-Irwin, Inc., 1974.
- Solomon and Pringle. *An Introduction to Financial Management*. Santa Monica, Calif.: Goodyear Publishing, 1977. Pages 261-62.

---

## Lease/Buy Decision

---

This program computes the present value of the cost to lease, and the present value of the cost to buy. Any difference between those amounts is the advantage of leasing or of buying. It is assumed that the asset would be financed over the same period of time that it would be leased.

To use the program, enter the price of the asset, the interest rate, the term in years, the salvage value at the end of that term, the tax rate, annual amount of loan payments, and the annual amount of lease payments. The program outputs the present value of the cost to buy, the present value of the cost to lease, and the difference between those amounts.

While this program may be instructive in pointing out decision factors you may have overlooked, it is not meant to replace your judgement. Capital planning requirements and lease/loan terms must ultimately guide your decision. In general, depreciation and salvage value reduce the cost of buying. However, if an asset is subject to rapid obsolescence, leasing may be the less expensive choice.

### Program Notes

This program is actually a modified version of the Net Present Value of an Investment program. As such, you may find it instructive of modifications you may make to any of the programs in this book.

### Example

Acme Landscaping has need for a small truck for everyday use. They are considering buying a truck for \$6,000. Salvage value after 4 years is estimated to be \$2,000. The bank will lend \$6,000 at 16 percent interest to be repaid in 4 equal installments of \$2,145. The lease will cost \$2,000 per year. Taxes are 40%, and straight-line depreciation of \$1,000 per year will be used. What is the present value of the cost to buy? What is the present value of the cost to lease? Should Acme lease or buy?

Answer: The present value of the loan is \$3,011.90. The present value of the lease is \$3,357.82. Acme should buy the truck.

### LEASE/BUY DECISION

```
ENTER THE COST TO ACQUIRE THE ASSET (PRINCIPAL OF THE LOAN)? 6000
ENTER THE INTEREST RATE (%)? 16
ENTER THE TERM IN YEARS? 4
WHAT IS THE SALVAGE VALUE AT THE END OF 4 YEARS? 2000
```

```
WHAT IS THE TAX RATE (%)? 40
ENTER THE ANNUAL AMOUNT OF LOAN PAYMENTS? 2145
ENTER THE ANNUAL AMOUNT OF LEASE PAYMENTS? 2000
```

```
ENTER THE DEPRECIATION AMOUNT FOR EACH YEAR.
```

```
YEAR NUMBER 1 ? 1000
              2 ? 1000
              3 ? 1000
              4 ? 1000
```

```
PRESENT VALUE OF COST OF LOAN = $ 3011.9
PRESENT VALUE OF COST OF LEASE = $ 3357.82
```



ADVANTAGE OF BUYING = \$ 345.92

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA (Y/N)? N

### Practice Problems

1. In the above example, what if the lease is \$1,200 per year?

Answer: Leasing would be the best choice. The present value of the lease would be \$2,014.69. The leasing advantage would be \$997.21.

2. Industrial Supply Company needs a computer for their in-house use. The model they want will cost \$30,000, to be financed at 17% interest over five years. After five years ISC plans to sell the computer for \$10,000 and buy a larger model. The tax rate is 48%, annual loan payments will be \$9,375.00, and a five year lease on the equipment would cost \$3,500.00 per year. Depreciation would be \$6,000.00 the first year, \$5,000 year 2, \$4,000 year three, \$3,000 year four, and \$2,000 year 5. What is the advantage of leasing or buying?

Answer: ISC would realize an advantage of \$7,362.24 if they lease the new computer.

### Program Listing

```
10 PRINT "LEASE/BUY DECISION"
20 REM - FUNCTION TO ROUND TO NEAREST HUNDREDTH
30 DEF FNA(X)=INT(X*100+.5)/100
40 PRINT
50 PRINT "ENTER THE COST TO ACQUIRE ASSET (PRINCIPAL OF LOAN)";
60 INPUT B1
70 PRINT "ENTER THE INTEREST RATE (%)";
80 INPUT I1
90 REM - CONVERT INTEREST RATE TO DECIMAL
100 I1=I1/100
110 PRINT "ENTER THE TERM IN YEARS";
120 INPUT Y1
130 PRINT "WHAT IS THE SALVAGE VALUE AT THE END OF";Y1;"YEARS";
140 INPUT S1
150 PRINT
160 PRINT "WHAT IS THE TAX RATE (%)";
170 INPUT R1
180 REM - CONVERT TAX RATE TO DECIMAL
190 R1=R1/100
200 PRINT "ENTER THE ANNUAL AMOUNT OF LOAN PAYMENTS";
210 INPUT A1
220 PRINT "ENTER THE ANNUAL AMOUNT OF LEASE PAYMENTS";
230 INPUT A2
240 REM - RESET TOTAL AMOUNTS TO ZERO
250 T1=0
260 L1=0
270 PRINT
280 PRINT "ENTER THE DEPRECIATION AMOUNT FOR EACH YEAR."
290 PRINT
300 REM - LOOP TO INPUT, CALCULATE, AND ACCUMULATE VALUES EACH
    YEAR
310 FOR Z=1 TO Y1
320 IF Z>1 THEN 350
330 PRINT "YEAR NUMBER";
340 GOTO 360
```

```

350 PRINT "
360 PRINT Z;
370 INPUT D1
380 REM - CALCULATE INTEREST AMOUNT FOR EACH YEAR
390 B0=ABS(B1-FNA(B1*(1+I1)))
400 REM - CONVERT D1 TO PRESENT VALUE OF COST OF OWNING EACH YEAR
410 D1=FNA((A1-FNA((D1+B0)*R1))/((1+I1)^Z))
420 REM - SUBTRACT ANNUAL PAYMENT, ADD ANNUAL INTEREST TO PRINCIPAL
430 B1=B1-A1+B0
440 REM - SUM PRESENT VALUE AMOUNTS OF EACH YEAR
450 T1=T1+D1
460 REM - COMPUTE PRESENT VALUE OF COST TO LEASE FOR EACH YEAR
470 L1=L1+FNA((A2-(A2*R1))/((1+I1)^Z))
480 NEXT Z
490 REM - SUBTRACT PRESENT VALUE OF SALVAGE VALUE FROM TOTAL COST
    TO OWN
500 T1=T1-FNA(S1/((1+I1)^Y1))
510 REM - OUTPUT RESULTS
520 PRINT
530 PRINT "PRESENT VALUE OF COST OF LOAN = $";T1
540 PRINT "PRESENT VALUE OF COST OF LEASE = $";L1
550 PRINT
560 IF L1<T1 THEN 590
570 PRINT "ADVANTAGE OF BUYING = $";L1-T1
580 GOTO 600
590 PRINT "ADVANTAGE OF LEASING = $";T1-L1
600 PRINT
610 REM - RESTART OR END PROGRAM?
620 PRINT "WOULD YOU LIKE TO RE-RUN PROGRAM WITH NEW DATA (Y/N)";
630 INPUT Z$
640 IF Z$="Y" THEN 40
650 IF Z$<>"N" THEN 620
660 END

```

## Reference

Chase and Aquilano. *Production and Operations Management*. Homewood, Ill.: Richard D. Irwin, Inc., 1977. Pages 138-40.

---

## Syndicated Investment Analysis

---

This program evaluates tax savings and net cashflows from an investment by a syndicate, or group of investors, to a participating investor. The program considers the investor's tax bracket, as well as the proportion of the original investment, participation in cash income, taxable income/loss, and tax credits.

To use this program, enter the length of the analysis in years and the first year of syndication. Then, for each year, enter the cash income for the syndicate, followed by its taxable income. Enter the year (1, 2, etc.) and total investment for that year by the syndicate. Then, enter the year and amount of investment or other tax credits (entered as a negative number), or credit recapture (entered as a positive number). Next, enter allocation percentages for the investor: percentage of total investment, cash, income, and taxable income (or loss) and credits. The final entry is the investor's tax bracket, entered as a percentage.

The program then prints its analysis, which shows the investor his/her original investment, cash income, taxable income, tax saving (tax savings are negative; tax paid is positive), net end-of-year cashflow and cumulative net cashflows. You may repeat the analysis for different tax brackets when the program asks for a new tax bracket to consider. (All other investment factors remain as you last entered them.) Enter a tax bracket of 999 to respecify the percentage allocations. Enter an investment allocation percentage of 999 to end the program.

### Program Notes

The program is set for 40 years of projections. You can change this amount by modifying lines 20 and 30 as follows:

```
20 N9 = 1
30 DIM C(I),D(I),J(I),K(I),S(I),T(I),U(I),V(I)
```

Make sure that you replace the expression I with a constant equal to the maximum number of years.

### Example

Consider this syndicated investment: An income property with a \$35,000 down payment which will generate \$4,500 cash over the first 4 years, \$5,200 over the next 4 years, and \$5,500 over the remaining 5 years. The investment earns a \$3,500 investment tax credit in the first year. Taxable income will start at -\$3,800 and increase by \$1,100 per year for the life of the investment.

The investor is in the 55 percent tax bracket, and is contributing 30 percent of the original cash outlay. Participation is 30 percent on cash income and taxable income. How will this investor run the program?

Answer: The printout below shows the investor's portion of cash income, tax savings, net and cumulative cashflow. At the end of the investment projection, cumulative cash to this investor is \$4,432, and the investment is sheltered until the end of 1985, where a tax of \$109 must be paid.

### SYNDICATED INVESTMENT ANALYSIS

```
FOR HOW MANY YEARS DO YOU WANT
THIS PROJECTION (LIMIT: 40 )? 13
```

```
ENTER THE FIRST YEAR OF
SYNDICATION (E.G. 1981)? 1980
```

```
FOR ENTIRE SYNDICATE, ENTER CASH INCOME
FOR EACH YEAR OF PROJECTION
YEAR 1 CASH INCOME=? 4500
```

YEAR 2 CASH INCOME=? 4500  
YEAR 3 CASH INCOME=? 4500  
YEAR 4 CASH INCOME=? 4500  
YEAR 5 CASH INCOME=? 5200  
YEAR 6 CASH INCOME=? 5200  
YEAR 7 CASH INCOME=? 5200  
YEAR 8 CASH INCOME=? 5200  
YEAR 9 CASH INCOME=? 5500  
YEAR 10 CASH INCOME=? 5500  
YEAR 11 CASH INCOME=? 5500  
YEAR 12 CASH INCOME=? 5500  
YEAR 13 CASH INCOME=? 5500

FOR ENTIRE SYNDICATE, ENTER TAXABLE  
INCOME FOR EACH YEAR OF PROJECTION  
POSITIVE FOR INCOME NEGATIVE FOR LOSS

YEAR 1 TAXABLE=?-3800  
YEAR 2 TAXABLE=?-2700  
YEAR 3 TAXABLE=?-1600  
YEAR 4 TAXABLE=?-500  
YEAR 5 TAXABLE=? 600  
YEAR 6 TAXABLE=? 1700  
YEAR 7 TAXABLE=? 2800  
YEAR 8 TAXABLE=? 3900  
YEAR 9 TAXABLE=? 5000  
YEAR 10 TAXABLE=? 6100  
YEAR 11 TAXABLE=? 7200  
YEAR 12 TAXABLE=? 8300  
YEAR 13 TAXABLE=? 9400

ENTER YEAR OF VENTURE (1, 2, ETC.) AND  
AMOUNT OF INVESTMENT BY ENTIRE GROUP  
OF INVESTORS THAT YEAR. AFTER LAST  
YEAR, ENTER 99999,0

? 1,35000  
? 99999,0

ENTER YEAR OF VENTURE (1, 2, ETC.) AND  
AMOUNT OF INVESTMENT CREDIT OR OTHER  
SIMILAR CREDIT FOR ENTIRE SYNDICATE  
(AS NEGATIVE), AND CREDIT RECAPTURE  
(AS POSITIVE) FIGURE. AFTER LAST  
ENTRY, ENTER 99999,0

? 1,-3500  
? 99999,0

ENTER PERCENTAGE ALLOCATIONS (0-100%)  
FOR THIS INVESTOR...

PCT. OF INVESTMENT (999=END)? 30  
PCT. OF CASH INCOME? 30

PCT. OF TAXABLE INCOME  
(OR LOSS), AND CREDITS? 30

ENTER TAX BRACKET (999=CHANGE ALLOCATIONS)? 55  
RESULTS FOR INVESTOR IN 0 % TAX BRACKET

YEAR	INVEST- MENT	CASH INCOME	TAX SAVING	NET CASH	CUMULATIVE CASH
1980	10500	1350	-1677	-7473	-7473
1981	0	1350	-446	1796	-5677
1982	0	1350	-264	1614	-4063
1983	0	1350	-82	1432	-2631
1984	0	1560	99	1461	-1170
1985	0	1560	281	1279	109
1986	0	1560	462	1098	1207
1987	0	1560	644	916	2123
1988	0	1650	825	825	2948
1989	0	1650	1007	643	3591
1990	0	1650	1188	462	4053
1991	0	1650	1370	280	4333
1992	0	1650	1551	99	4432

THIS SCHEDULE DISREGARDS MINIMUM TAX,  
DISALLOWANCE OF INVESTMENT INTEREST EXPENSE,  
CODE SEC. 183, ETC.

### Practice Problems

1. Alvin wants to start a musical career with his brothers Simon and Theodore. Alvin is in the 40 percent tax bracket. He will contribute 45 percent of the \$30,000 needed to build a recording studio. He will participate 20 percent in the cash earnings, and 45 percent in the taxable earnings of the company. Alvin expects that the studio will generate \$8,000 cash per year for the first 2 years. A further investment of \$15,000 will come up in year 3 for new equipment. The studio's taxable earnings will start at -\$4,200, increasing by \$1,000 each year. Cash income for the recording studio will increase to \$12,000 per year from year 3 to year 10 (the last year of projection).

What will Alvin's cumulative cashflow be from this investment? In what year will Alvin have to start paying taxes on his share of the investment? Assume that the studio will earn a 10 percent investment tax credit for the initial cash outlay as well as the \$15,000 in year 3.

Answer: Alvin's cumulative cashflow will be \$3,635 at the end of year 10. Assuming the first year is 1980, Alvin will have to start paying taxes on this investment in 1985 (\$144).

2. Fred wants to start a helicopter tour service. He is in the 65 percent tax bracket, and will participate in all aspects of the syndicate at 51 percent. The initial investment for a 4-passenger helicopter is \$12,500. Fred plans on trading up to a 6-passenger helicopter after 3 years. The group will receive a \$6,500 tax credit in year 1. If they trade up in year 3, they will receive an \$8,500 tax credit, and will have to invest another \$19,000. They will sell the 4-passenger helicopter in year 4, losing \$4,167 from credit recapture. Cash income will start at \$40,000 per year, growing to \$48,000 per year at the start of year 3, up until year 8 (the final year of projection). Taxable income starts at -\$9,000, growing by \$2,000 every year.

What will the total cumulative cashflow be for the 8 years of projection? How will the credit recapture affect him in year 4?

Answer: Total cumulative cashflow will be \$182,441. Fred will have to pay \$1,131 in taxes in year 4, due to the credit recapture.

### Program Listing

```
1 PRINT "SYNDICATED INVESTMENT ANALYSIS"
2 PRINT
9 REM ROUND-OFF FUNCTION
10 DEF FNR(X)=INT(X+.5)
18 REM N9=MAX YRS FOR PROJECTION
19 REM    AND MAX DIMENSION FOR LINE 30
20 N9=40
30 DIM C(40),J(40),T(40),U(40)
200 PRINT "FOR HOW MANY YEARS DO YOU WANT"
210 PRINT "THIS PROJECTION (LIMIT: ";N9;")";
220 INPUT Y
225 IF Y>N9 THEN 200
230 PRINT
240 PRINT "ENTER THE FIRST YEAR OF"
250 PRINT "SYNDICATION (E.G. 1981)";
260 INPUT Y1
270 PRINT
280 PRINT "FOR ENTIRE SYNDICATE, ENTER CASH INCOME"
290 PRINT "FOR EACH YEAR OF PROJECTION"
300 FOR I=1 TO Y
310 PRINT "YEAR";I;"CASH INCOME=";
320 INPUT C(I)
340 NEXT I
350 PRINT
360 PRINT "FOR ENTIRE SYNDICATE, ENTER TAXABLE"
370 PRINT "INCOME FOR EACH YEAR OF PROJECTION"
380 PRINT "POSITIVE FOR INCOME NEGATIVE FOR LOSS"
390 FOR I=1 TO Y
400 PRINT "YEAR";I;"TAXABLE=";
410 INPUT T(I)
430 NEXT I
440 PRINT
450 PRINT "ENTER YEAR OF VENTURE (1, 2, ETC.) AND"
460 PRINT "AMOUNT OF INVESTMENT BY ENTIRE GROUP"
470 PRINT "OF INVESTORS THAT YEAR. AFTER LAST"
480 PRINT "YEAR, ENTER 99999,0"
490 INPUT I,X0
500 IF I=99999 THEN 530
505 J(I)=X0
520 GO TO 490
530 PRINT
540 PRINT "ENTER YEAR OF VENTURE (1, 2, ETC.) AND"
550 PRINT "AMOUNT OF INVESTMENT CREDIT OR OTHER"
560 PRINT "SIMILAR CREDIT FOR ENTIRE SYNDICATE"
570 PRINT "(AS NEGATIVE), AND CREDIT RECAPTURE"
580 PRINT "(AS POSITIVE) FIGURE. AFTER LAST"
590 PRINT "ENTRY, ENTER 99999,0"
600 INPUT I,X0
610 IF I=99999 THEN 640
615 U(I)=X0
630 GO TO 600
```

```

640 PRINT
645 PRINT "ENTER PERCENTAGE ALLOCATIONS (0-100%)"
650 PRINT "FOR THIS INVESTOR..."
655 PRINT "PCT. OF INVESTMENT (999=END)";
660 INPUT P1
665 IF P1>998 THEN 2170
670 P1=P1/100
675 PRINT "      PCT. OF CASH INCOME";
680 INPUT P2
685 P2=P2/100
690 PRINT "      PCT. OF TAXABLE INCOME"
693 PRINT "      (OR LOSS), AND CREDITS";
695 INPUT P3
700 P3=P3/100
705 PRINT
710 PRINT "ENTER TAX BRACKET (999=CHANGE ALLOCATIONS)";
720 INPUT T1
725 IF T1>998 THEN 640
750 PRINT "RESULTS FOR INVESTOR IN";T1;"% TAX BRACKET"
755 T1=T1/100
760 PRINT
770 PRINT "YEAR","INVEST-","CASH","TAX"," NET","CUMULATIVE"
780 PRINT, " MENT","INCOME","SAVING","CASH"," CASH"
800 PRINT
810 S1=0
820 FOR I=1 TO Y
850 K=FNR(P1*J(I))
870 D=FNR(P2*C(I))
890 V=FNR(P3*T(I)*T1+P3*U(I))
900 S=D-K-V
910 S1=S1+S
920 PRINT Y1+I-1,K,D,V,S,S1
940 IF I/3<>INT(I/3) THEN 960
950 PRINT
960 NEXT I
2099 REM PRINT DISCLAIMER/BLANK LINES
2100 PRINT "THIS SCHEDULE DISREGARDS MINIMUM TAX,"
2110 PRINT "DISALLOWANCE OF INVESTMENT INTEREST EXPENSE,"
2120 PRINT "CODE SEC.183, ETC."
2130 PRINT
2140 PRINT
2150 PRINT
2160 GOTO 710
2170 END

```

---

## Depreciation Switch

---

An accelerated depreciation method provides for greatest depreciation in the earlier years. At some point, switching to a straight-line depreciation will allow a larger amount to be depreciated in later years than could be done by continuing to use the accelerated method.

Calculations are made using a fixed cost of 1 million dollars. The actual cost of the asset involved is unimportant. The million dollar cost serves only to separate close calculations. Enter the depreciation method to use for this asset, in percent (i.e., 125, 150, 200, etc.); the useful life of the asset, in years; and the number of months of depreciation the first year of the useful life (a full first year should be entered as 12 months).

### Example

Champion Products acquired a plastic injection machine that has a useful life of 5 years. Six months' depreciation remains in this fiscal year, and Champion plans to use 200% declining balance depreciation. When should they switch from declining balance method to straight-line depreciation in order to maximize the amounts depreciated?

Answer: Champion should switch methods in the 5th year.

### DEPRECIATION SWITCH

```
ENTER METHOD, IN PERCENT (0=END)? 200
ENTER USEFUL LIFE OF ASSET, IN YEARS? 5
ENTER NUMBER OF MONTHS DEPRECIATION LEFT IN FIRST YEAR? 6

YEAR OF SWITCH = 5
```

```
ENTER METHOD, IN PERCENT (0=END)? 0
```

### Practice Problems

1. In the above example, what if 12 months depreciation remains in the current fiscal year?

Answer: The switch should be effected in the 4th year.

2. Using 150% depreciation, when should an asset with an 8 year life be depreciated by the straight-line method, assuming a full year's depreciation remains in the first year?

Answer: The switch to straight-line should be made in the 4th year.



## Program Listing

```
10 PRINT "DEPRECIATION SWITCH"
20 REM - USE MILLION DOLLAR COST TO
30 REM - SEPERATE CLOSE CALCULATIONS
40 C=1E+6
50 REM - RESET ACCUMULATED DEPRECIATION TO ZERO
60 A=0
70 PRINT
80 PRINT "                ENTER METHOD, IN PERCENT (0=END)";
90 INPUT T
100 IF T=0 THEN 350
110 T=T/100
120 PRINT "                ENTER USEFUL LIFE OF ASSET, IN YEARS";
130 INPUT L
140 IF L<=3 THEN 170
150 PRINT "LIMIT 3 YEARS MINIMUM LIFE, PLEASE RE-ENTER."
160 GOTO 120
170 PRINT "ENTER NUMBER OF MONTHS DEPRECIATION LEFT IN FIRST YEAR";
180 INPUT M
190 Y=1
200 REM - CALCULATE DEPRECIATION ACCUMULATED IN THE FIRST YEAR
210 A=INT(((M/12)*(T/L)*C)*100+.5)/100
220 Y=Y+1
230 REM - COMPUTE AMOUNT OF DEPRECIATION THIS YEAR
240 D=INT(((T/L)*(C-A))*100+.5)/100
250 REM - IF DEPRECIATION IS LESS THAN BOOK VALUE DIVIDED BY
260 REM - REMAINING LIFE, PRINT YEAR NUMBER
270 IF D<(C-A)/(L-Y+1+(12-M)/12) THEN 310
280 REM - IF NOT, INCREMENT ACCUMULATED DEPRECIATION
290 A=A+D
300 GOTO 220
310 PRINT
320 PRINT "                YEAR OF SWITCH =";Y
330 PRINT
340 GOTO 60
350 END
```

## References

U.S. Internal Revenue Service Code, Section 167(b) and Section 167(e)(1).

U.S. Treasury Department, Internal Revenue Service. Regulations, Sections 1.167(b)-0, 1.167(b)-1, 1.167(b)-2, and 1.167(e)-1.

---

## Apportionment By Ratios

---

This program divides a quantity into the proportion that each of a group of numbers bears to the sum of that group. You are first asked for the number of decimal places that you wish shown from whole numbers down to thirteen decimal places (if your computer is that accurate). You then enter the value to be apportioned, and the number of parts into which it is to be divided. You then enter each component of the group to be used as the basis for apportionment. The program prints out a table that shows each of these amounts, the percentage each is of the group total, and the corresponding apportioned amount. At the conclusion, it prints the totals of these three columns.

### Example

Ten employees at Widgets, Inc. are receiving bonuses from a \$30,000 pool. If each receives a share proportionate to his salary, how much does each one get?

Name	Salary
Abelson	\$54,000
Boucher	\$47,000
Charleston	\$40,000
Dryden	\$33,500
Evans	\$29,750
Freisner	\$26,000
Goodine	\$24,500
Holloway	\$21,000
Ishikawa	\$17,500
Johnson	\$15,000

Answer:

```
APPORTIONMENT BY RATIOS
ENTER THE NUMBER OF DECIMAL
PLACES OF ROUNDING YOU WANT:
0 FOR WHOLE NUMBERS, 1 FOR TENTHS, ETC.
UP TO THIRTEEN (IF HARDWARE PERMITS.)
? 2
ENTER TOTAL TO BE APPORTIONED
? 30000
ENTER NUMBER OF PORTIONS
? 10
ENTER AMOUNT 1
? 54000
ENTER AMOUNT 2
? 47000
ENTER AMOUNT 3
? 40000
ENTER AMOUNT 4
? 33500
ENTER AMOUNT 5
? 29750
ENTER AMOUNT 6
? 26000
ENTER AMOUNT 7
```

? 24500  
 ENTER AMOUNT 8  
 ? 21000  
 ENTER AMOUNT 9  
 ? 17500  
 ENTER AMOUNT 10  
 ? 15000

	AMOUNT	PERCENT	APPORTIONED
	54000	17.52	5255.47
	47000	15.25	4574.21
	40000	12.98	3892.94
	33500	10.87	3260.34
	29750	9.65	2895.38
	26000	8.43	2530.41
	24500	7.95	2384.43
	21000	6.81	2043.8
	17500	5.68	1703.16
	15000	4.86	1459.86
TOTALS	308250	100.00	30000

LAST ITEM ADJUSTED WHERE NECESSARY

### Practice Problems

1. A mayor running for re-election wants to divide his campaign workers among the city's six districts based on the population of each district. He has 42 campaign workers, and the districts are populated as follows: District 1: 29,842; District 2: 17,420; District 3: 14,625; District 4: 24,314; District 5: 21,209; District 6: 18,956. How many workers should he place in each district?

Answer: District 1: 10; District 2: 6; District 3: 5; District 4: 8; District 5: 7; District 6: 6.

2. A winery has 120 bottles of wine that it wants to distribute among its employees. If the wine is divided in proportion to each employee's seniority, how much wine does each employee get?

Name	Years Employed
Jones	22
Romero	18
Lippitt	14
Doyle	8
Peterson	4
Covey	2
Miller	2
Bennett	1

Answer: Jones: 37 bottles; Romero: 30 bottles; Lippitt: 24 bottles; Doyle: 14 bottles; Peterson: 7 bottles; Covey: 3 bottles; Miller: 3 bottles; Bennett: 2 bottles.

### Program Listing

```

10 PRINT "APPORTIONMENT BY RATIOS"
20 DIM A(100)
30 PRINT "ENTER THE NUMBER OF DECIMAL"
40 PRINT "PLACES OF ROUNDING YOU WANT:"
50 PRINT "0 FOR WHOLE NUMBERS, 1 FOR TENTHS, ETC."
60 PRINT "UP TO THIRTEEN (IF HARDWARE PERMITS.)"
70 INPUT R1
80 PRINT "ENTER TOTAL TO BE APPORTIONED"
90 INPUT S2

```

```

100 PRINT "ENTER NUMBER OF PORTIONS"
110 INPUT N
119 REM ENTER RATIO AMOUNTS ONE BY ONE
120 FOR I=1 TO N
130 PRINT "ENTER AMOUNT";I
140 INPUT A(I)
150 S1=S1+A(I)
160 NEXT I
170 PRINT , "AMOUNT", "PERCENT", "APPORTIONED"
180 PRINT
189 REM PRINT OUT TABLE
190 FOR I=1 TO N-1
200 P=INT(10000*A(I)/S1+.5)/100
210 P1=P1+P
220 R=INT((S2*A(I)/S1)*10^(R1)+.5)/10^(R1)
230 S3=S3+R
240 PRINT , A(I), P, R
250 NEXT I
260 PRINT , A(N), 100-P1, S2-S3
270 PRINT
280 PRINT "TOTALS", S1, "100.00", S2
290 PRINT
300 PRINT "LAST ITEM ADJUSTED WHERE NECESSARY"
310 END

```

---

## Internal Rate of Return

---

Internal Rate of Return (IRR) is the rate at which the sum of all cash flows discount to the amount of the initial investment. This program finds the rate by using a half-interval search.

To use the program, enter the amount of the initial investment, then the term of the investment (in years), and the cash flow amount for each year. Enter outflows (funds you invest) as negative numbers. Enter an initial investment of zero to end the program.

IRR can also be used to compute the yield to maturity of a bond by entering the price of the bond as the initial investment, the number of years to maturity as the term, coupon amounts for each year they will be received as the cash flow amounts for those years (enter the total amount to be received in each year), and coupon amount(s) plus the maturity value of the bond in the last year (when the bond will mature). The IRR returned by the program is the yield to maturity of the bond.

### Program Notes

The half-interval search at lines 320 to 540 will find rates of return between 0% and 99%. If this range is not wide enough to suit your needs, change the initial values of variable L at line 330 and H at line 340. These are the low and high search limits. Make sure that upon the first execution of line 370, the value of  $(L+H)/2$  is not zero, as that will cause premature exit from the search algorithm.

### Example

Bob T. has an opportunity to invest in a venture. An initial investment of \$10,000 is needed, with cash returns of \$4,000, \$5,000, and \$3,000 over the next 3 years. His required rate of return is 15%. Should Bob accept this investment?

Answer: No. The IRR of this investment is 10.1331%. The accept/reject criterion stipulates rejection of any investment whose IRR is less than the required rate of return.

INTERNAL RATE OF RETURN

ENTER THE AMOUNT OF THE INITIAL INVESTMENT (0 TO END)? 10000

NUMBER OF CASH FLOW PERIODS? 3

(ENTER INFLOWS AS POSITIVE, OUTFLOWS AS NEGATIVE AMOUNTS)

CASH FLOW FOR PERIOD 1 ? 4000

2 ? 5000

3 ? 3000

INTERNAL RATE OF RETURN = 10.1331 %

ENTER THE AMOUNT OF THE INITIAL INVESTMENT (0 TO END)? 0

### Practice Problem

A new bond issue offers a coupon rate of 8.25% and matures in 7 years. What is the yield to maturity of a \$10,000 bond if the price is \$8,500?

Answer: The yield to maturity is 11.4831%.

### Program Listing

```
10 PRINT "INTERNAL RATE OF RETURN"
20 REM FUNCTION TO ROUND TO NEAREST HUNDREDTH
30 DEF FNA(X)=INT(X*100+.5)/100
40 REM FUNCTION TO ROUND TO NEAREST TEN-THOUSANDTH
50 DEF FNB(X)=INT(X*1E5+.5)/1E5
60 REM CHANGE SIZE OF ARRAY C() IF NECESSARY
70 DIM C(12)
80 PRINT
90 PRINT "ENTER THE AMOUNT OF THE INITIAL INVESTMENT (0 TO END)";
100 INPUT I
110 REM END PROGRAM?
120 IF I=0 THEN 590
130 PRINT
140 PRINT "                                NUMBER OF CASH FLOW PERIODS";
150 INPUT N
160 REM RESTART IF NUMBER OF CASH FLOW PERIODS IS INVALID
170 IF N<1 THEN 80
180 REM LOOP TO INPUT AND SUM CASH FLOW AMOUNT(S)
190 F=0
200 PRINT
210 PRINT "(ENTER INFLOWS AS POSITIVE, OUTFLOWS AS NEGATIVE AMOUNTS)"
220 FOR J=1 TO N
230 IF J>1 THEN 260
240 PRINT "                                CASH FLOW FOR PERIOD";
250 GOTO 270
260 PRINT "                                ";
270 PRINT J;
280 INPUT C(J)
290 F=F+FNA(C(J))
300 NEXT J
310 PRINT
320 REM INITIALIZE VALUES
330 L=0
340 H=1
350 R1=0
360 REM GUESS RATE = (HIGH RATE + LOW RATE) / 2
370 R=(L+H)/2
380 REM EXIT IF RATE REMAINS UNCHANGED
390 IF R=R1 THEN 550
400 REM SET LAST GUESS TO CURRENT GUESS
410 R1=R
420 REM ADD PRESENT VALUES FOR EACH YEAR BASED ON RATE OF R
430 T=0
440 FOR J=1 TO N
450 T=T+FNA(C(J)/((R+1)^J))
460 NEXT J
```

```
470 REM IF TOTAL PRESENT VALUES EQUAL INVESTMENT, EXIT
480 IF T=I THEN 550
490 REM SET HIGH OR LOW RATE TO CURRENT GUESS
500 IF I>T THEN 530
510 L=R
520 GOTO 370
530 H=R
540 GOTO 370
550 PRINT
560 PRINT "INTERNAL RATE OF RETURN =";FNB(R*100);"%"
570 PRINT
580 GOTO 80
590 END
```

## References

- Chase and Aquilano. *Production and Operations Management*. Homewood, Ill.: Richard D. Irwin, Inc., 1977. Pages 131-32.
- Rosen, Lawrence R. *The Dow Jones-Irwin Guide to Interest*. Homewood, Ill.: Dow Jones-Irwin, 1974.
- Solomon and Pringle. *An Introduction to Financial Management*. Santa Monica, Calif.: Goodyear Publishing, 1977. Pages 257-61.

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## Financial Management Rate of Return

---

Financial Management Rate of Return (FMRR) differs from Internal Rate of Return in several respects. For some investments, particularly real estate ventures, FMRR will provide a more realistic value than IRR will. FMRR assumes only cash flows after financing and taxes are considered, and it ignores the fact that other sources of funds may be available.

To use the program, you enter the term of the investment (in years), then a liquid investment rate. This is a rate at which funds can be invested in any amount, at a guaranteed after-tax rate, and withdrawn as needed (such as a savings account). You also enter a 'safe' fixed investment rate. 'Safe' means the return on the investment will be at least that high. This investment can be a real estate project or other fixed investment of comparable risk at after-tax rates above the liquid rate, such as certificates of deposit or Treasury bills. The fixed investment should have a minimum amount that can be invested. Enter this amount too.

The program will indicate points where you will be expected to invest funds in the liquid and fixed investments, the actual initial investment you will need to make (the difference between that amount and the original initial investment must be invested at the fixed rate at the beginning of the first year), the actual total return on the investment, and the rate at which the actual total return discounts to the actual initial investment (the FMRR).

### Example

Horatio plans to buy an apartment house. The terms require \$10,000 down payment to be made now, and payments of \$50,000 to be made next year and the following year. Cash flows indicate that at the end of years 3 and 5, Horatio can expect to receive \$30,000 from his investment. He plans to remodel the building during year 4, at an estimated cost of \$20,000. Finally, in year 6 he plans to sell the building for \$250,000. The liquid investment rate available is 5%, and a minimum \$10,000 fixed investment will earn at least 10%. What is the FMRR on Horatio's investment?

Answer: 19.348%. (The IRR of this investment is 25.2%.)

'FINANCIAL MANAGEMENT' RATE OF RETURN

NUMBER OF YEARS? 6

LIQUID INVESTMENT INTEREST RATE? 5

'SAFE' FIXED INVESTMENT INTEREST RATE? 10

MINIMUM AMOUNT OF FIXED INVESTMENT? 10000

(ENTER INFLOW AMOUNTS AS POSITIVE VALUES, OUTFLOWS AS NEGATIVE.)

ENTER CASH FLOW AMOUNT FOR YEAR 0 ? -10000

1 ? -50000

2 ? -50000

3 ? 30000

4 ? -20000

5 ? 30000

6 ? 250000



LIQUID INVESTMENT OF \$ 19047 TO BE MADE AT END OF YEAR 3

FIXED INVESTMENT OF \$ 10952 TO BE MADE AT END OF YEAR 3

FIXED INVESTMENT OF \$ 30000 TO BE MADE AT END OF YEAR 5

ACTUAL TOTAL INITIAL INVESTMENT = \$ 102971

TOTAL RETURN ON INVESTMENT = \$ 297577

'FINANCIAL MANAGEMENT' RATE OF RETURN = 19.348 %

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA? (Y/N)? N

### Practice Problems

1. What is the FMRR on a 6 year project if the liquid rate is 7.25%, the fixed rate is 15% (with a minimum investment of \$10,000), and the initial investment is \$100,000? Cash flows will be \$30,000 inflow year 1, \$45,000 outflow year 2, and \$50,000 inflows during each of the remaining 4 years of the term.

Answer: The FMRR is 11.783%.

2. On a 4 year investment, requiring \$10,000 initially and cash flows of -\$2,500, \$5,000, -\$2,500 and \$25,000 during the term, what is the FMRR? The liquid rate is 8.5%, and a minimum \$1,000 fixed investment will return at least 13%.

Answer: The FMRR is 23.303%.

### Program Listing

```
10 PRINT "'FINANCIAL MANAGEMENT' RATE OF RETURN"
20 REM FUNCTION TO ROUND TO NEAREST THOUSANDTH
30 DEF FNB(X)=INT(X*1E3+.5)/1E3
40 REM -- CHANGE DIMENSION OF ARRAY C() TO MAXIMUM NUMBER OF YEARS
50 DIM C(12)
60 PRINT
70 PRINT "NUMBER OF YEARS";
80 INPUT N
90 PRINT "LIQUID INVESTMENT INTEREST RATE";
100 INPUT R1
110 R1=R1/100+1
120 PRINT "'SAFE' FIXED INVESTMENT INTEREST RATE";
130 INPUT R2
140 R2=R2/100+1
150 PRINT "MINIMUM AMOUNT OF FIXED INVESTMENT";
160 INPUT M
170 PRINT
180 PRINT "(ENTER INFLOWS AS POSITIVE, OUTFLOWS AS NEGATIVE.)"
190 PRINT
200 PRINT "ENTER CASH FLOW AMOUNT FOR YEAR 0 ";
210 INPUT C0
220 FOR J=1 TO N
230 PRINT "
";J;
240 INPUT C(J)
250 NEXT J
260 PRINT
270 REM REMOVE ALL FUTURE OUTFLOWS BY UTILIZING
```

```

280 REM PRIOR INFLOWS WHERE POSSIBLE
290 REM
300 REM FIRST, FIND OUTFLOWS
310 FOR J=1 TO N-1
320 REM SKIP OVER INFLOWS AND ZERO AMOUNTS
330 IF C(J)>=0 THEN 520
340 REM OUTFLOW FOUND
350 A=C(J)
360 REM NOW FIND PRIOR INFLOW(S)
370 K=0
380 K=K+1
390 IF K=J THEN 520
400 IF C(J-K)<=0 THEN 380
410 REM INFLOW FOUND, REMOVE AMOUNT NEEDED TO ZERO OUTFLOW IF
    POSSIBLE
420 C(J-K)=C(J-K)+INT(A/R1^K)
430 IF C(J-K)>=0 THEN 490
440 REM IF NOT ENOUGH MONEY AVAILABLE, CORRECT TO ZERO THE INFLOW
450 A=A+INT(ABS(C(J-K))*R1^K)
460 C(J-K)=0
470 C(J)=A
480 GOTO 500
490 C(J)=0
500 PRINT "LIQUID INVESTMENT OF $";INT(ABS(A/R1^K));
510 PRINT "TO BE MADE AT END OF YEAR";J-K
520 NEXT J
530 PRINT
540 REM DISCOUNT REMAINING OUTFLOWS TO PRESENT AT LIQUID INTEREST
    RATE
550 FOR J=1 TO N-1
560 IF C(J)>=0 THEN 590
570 CO=CO+FNB(C(J)/R1^J)
580 C(J)=0
590 NEXT J
600 CO=INT(ABS(CO)+.5)
610 REM COMPOUND FORWARD ALL REMAINING INFLOWS GREATER
620 REM THAN THE MINIMUM FIXED INVESTMENT AMOUNT
630 FOR J=1 TO N-1
640 IF C(J)<M THEN 670
650 C(N)=C(N)+FNB(C(J)*R2^(N-J))
660 PRINT "FIXED INVESTMENT OF $";C(J);"TO BE MADE AT END OF YEAR"
    ;J
670 NEXT J
680 PRINT
690 C(N)=INT(ABS(C(N))+.5)
700 PRINT "ACTUAL TOTAL INITIAL INVESTMENT = $";CO
710 PRINT "TOTAL RETURN ON INVESTMENT = $";C(N)
720 REM INITIALIZE LOW AND HIGH GUESSES, SET LAST GUESS TO ZERO
730 L=0
740 H=1
750 R0=0
760 R=(H+L)/2
770 REM EXIT IF RATE REMAINS UNCHANGED
780 IF R=R0 THEN 910
790 REM SET LAST GUESS TO CURRENT GUESS
800 R0=R

```

```

810 REM CALCULATE PRESENT VALUE OF FUTURE VALUE BASED ON RATE OF R
820 T=INT(C(N)/((R+1)^N))
830 REM IF PRESENT VALUE EQUALS INVESTMENT, EXIT
840 IF T=C0 THEN 910
850 IF T>C0 THEN 890
860 REM SET HIGH OR LOW GUESS TO CURRENT GUESS
870 H=R
880 GOTO 760
890 L=R
900 GOTO 760
910 PRINT
920 PRINT "'FINANCIAL MANAGEMENT' RATE OF RETURN =";FNB(R*100);"%
930 REM RESTART OR END PROGRAM?
940 PRINT
950 PRINT "WOULD YOU LIKE TO RE-RUN PROGRAM WITH NEW DATA? (Y/N)";
960 INPUT Z$
970 IF Z$="Y" THEN 60
980 IF Z$<>"N" THEN 950
990 END

```

## References

- Determination and Usage of FM Rate of Return.* Detroit: Realtron Corporation, 1973.
- Messner, Schreiber, and Lyon. *Marketing Investment Real Estate Finance Taxation Techniques.* Chicago: Realtors National Marketing Institute of the National Association of Realtors, 1975.

## Financial Statement Ratio Analysis

This program calculates 22 ratios of interest to an investor, based on data you enter from a firm's financial statements. They indicate a firm's profitability, liquidity, activity and capital structure. You should only compare the ratios of a firm with others in the same industry, or against an industry average.

To use the program, enter the name of the firm which you are analyzing, the date of the financial statement and selected dollar amounts from it. You also need to enter the number of common shares outstanding, market price per share and dividends paid per share.

### Example

Jim would like to invest in an issue of common stock from a manufacturer of computer equipment. Its financial statements are shown below. Wimpytron has 7000 shares of common stock outstanding at a market price of \$17.50 per share. Dividends of \$1.25 per share were paid to stockholders of record from July 1979 through June 1980.

WIMPYTRON, Inc. Balance Sheet as of July 1, 1980 (figures in thousands of dollars)			
ASSETS		LIABILITIES AND EQUITY	
Cash	\$ 50	Accounts Payable	\$ 75
Accounts Receivable	100	Notes Payable	155
Marketable Securities	20	Total Current Liabilities	\$230
Inventory	200		
Total Current Assets	\$370	Long-Term Debt	190
Plant and Equipment	500	STOCKHOLDERS' EQUITY	
Less: Depreciation	30	Common Stock	40
Total Fixed Assets	470	Retained Earnings	380
			420
TOTAL ASSETS	<u>\$840</u>	TOTAL LIABILITIES AND EQUITY	<u>\$840</u>

WIMPYTRON, Inc. Income Statement as of July 1, 1980 (figures in thousands of dollars)	
Net Sales	\$985
Cost of Goods Sold	
Beginning Inventory	\$380
Purchases	200
Less: Ending Inventory	200
Total Cost of Goods Sold	<u>380</u>
Gross Margin	\$605
Selling Expenses	150
General & Administrative Expenses	220
Interest Expense	70
Total Expenses	440
Income Before Taxes	155
Income Taxes	78
Net Earnings After Taxes	<u>\$ 73</u>

How would you run the program to analyze this firm?

Answer:

## FINANCIAL STATEMENT RATIO ANALYSIS

NAME OF FIRM? WIMPYTRON INC.  
MONTH/DAY/YEAR? JULY 1 1980

### -----INCOME STATEMENT-----

ENTER AMOUNTS FOR:

NET SALES? 985000  
BEGINNING INVENTORY? 380000  
ENDING INVENTORY? 200000  
COST OF GOODS SOLD? 380000  
INTEREST EXPENSE? 70000  
PRE-TAX INCOME? 155000  
INCOME TAXES? 78000

### -----BALANCE SHEET-----

ENTER AMOUNTS FOR:

CASH? 50000  
ACCOUNTS RECEIVABLE? 100000  
NOTES & MARKETABLE SECURITIES? 20000  
TOTAL ASSETS? 840000  
CURRENT LIABILITIES? 230000  
STOCKHOLDERS' EQUITY? 420000

ALSO ENTER:

COMMON SHARES OUTSTANDING? 7000  
MARKET PRICE PER SHARE? 17.5  
DIVIDENDS PER SHARE? 1.25

### -----EVALUATION OF WIMPYTRON INC. BY RATIO ANALYSIS-----

PERIOD ENDING: JULY 1 1980

#### -----PROFITABILITY-----

RETURN ON ASSETS 9.2 %  
RETURN ON EQUITY 18.3 %  
RETURN ON INVESTED CAPITAL 12.6 %  
EARNINGS PER SHARE \$ 11  
OPERATING RATIO .843 :1

#### -----LIQUIDITY-----

NET WORKING CAPITAL \$ 140000  
ACID TEST (QUICK) RATIO .739 :1  
CURRENT RATIO 1.609 :1

#### -----ACTIVITY-----

SALES PER DAY \$ 2698.63  
DAYS SALES OUTSTANDING 37.056 DAYS  
INVENTORY TURNOVER 1.31 TIMES

#### -----INDEBTEDNESS-----

CREDITORS' INTEREST IN FIRM 50 %  
TIMES INTEREST EARNED 4.329  
DEBT TO EQUITY 1 :1  
LONG-TERM DEBT TO NET WORTH .452 :1  
LONG-TERM DEBT TO CAPITAL .311 :1

```

-----EQUITY-----
STOCKHOLDERS' INTEREST IN FIRM 50 %
      PAYOUT RATIO .114 :1
      EARNINGS YIELD 62.9 %
      BOOK VALUE/SHARE $ 60
      PRICE/EARNINGS RATIO 1.591 :1
      DIVIDEND YIELD 7.1 %

```

DO YOU WANT ANOTHER ANALYSIS (Y/N)? N

### Practice Problems

1. Suppose the balance sheet is altered as follows: long-term debt is \$220,000 and the stockholders' equity is \$390,000. What ratios will change, and what will their new values be?

Answer: Return on equity, 19.7%; Creditors' interest, 53.6%; debt to equity, 1.154:1; long-term debt to net worth, .564:1; long-term debt to capital, .361:1; stockholders' interest, 46.4%; payout ratio, .077:1; book value, \$55.714; dividend yield, 4.9%.

2. If you interchange the amounts for accounts receivable and cash, what ratios will change and what will their new values be?

Answer: Days sales outstanding changes to 18.528 days. All other ratios remain unchanged.

### Program Listing

```

10 PRINT "FINANCIAL STATEMENT RATIO ANALYSIS"
20 DIM D(20)
30 REM
40 REM D(1)   = NET SALES
50 REM D(2)   = BEGINNING INVENTORY
60 REM D(3)   = ENDING INVENTORY
70 REM D(4)   = COST OF GOODS SOLD
80 REM D(5)   = INTEREST EXPENSE
90 REM D(6)   = PRETAX INCOME
100 REM D(7)  = TAXES
110 REM D(8)  = CASH
120 REM D(9)  = ACCOUNTS RECEIVABLE
130 REM D(10) = NOTES RECEIVABLE
140 REM D(11) = TOTAL ASSETS
150 REM D(12) = CURRENT LIABILITIES
160 REM D(13) = EQUITY
170 REM D(14) = SHARES OUTSTANDING
180 REM D(15) = MARKET PRICE PER SHARE
190 REM D(16) = DIVIDENDS PAID
200 REM
210 DATA "NET SALES","BEGINNING INVENTORY","ENDING INVENTORY"
220 DATA "COST OF GOODS SOLD","INTEREST EXPENSE"
230 DATA "PRE-TAX INCOME","INCOME TAXES","CASH"
240 DATA "ACCOUNTS RECEIVABLE","NOTES & MARKETABLE SECURITIES"
250 DATA "TOTAL ASSETS","CURRENT LIABILITIES"
260 DATA "STOCKHOLDERS' EQUITY","COMMON SHARES OUTSTANDING"
270 DATA "MARKET PRICE PER SHARE","DIVIDENDS PER SHARE"
280 PRINT
290 PRINT "  NAME OF FIRM";
300 INPUT N$

```

```

310 PRINT " MONTH/DAY/YEAR";
320 INPUT D$
330 REM ENTER INCOME STATEMENT ACCOUNTS
340 RESTORE
350 PRINT "-----INCOME STATEMENT-----"
360 PRINT "ENTER AMOUNTS FOR:"
370 FOR I=1 TO 7
380 GOSUB 1620
390 NEXT I
400 REM ENTER BALANCE SHEET ACCOUNTS
410 PRINT "-----BALANCE SHEET-----"
420 PRINT "ENTER AMOUNTS FOR:"
430 FOR I=8 TO 13
440 GOSUB 1620
450 NEXT I
460 PRINT
470 PRINT "ALSO ENTER:"
480 FOR I=14 TO 16
490 GOSUB 1620
500 NEXT I
510 PRINT
520 PRINT "-----EVALUATION OF ";N$;" BY RATIO ANALYSIS-----"
530 PRINT "","PERIOD ENDING:";D$
540 PRINT
550 PRINT "","-----PROFITABILITY-----"
560 T$="RETURN ON ASSETS"
570 X1=2
580  $X0=(D(6)-D(7))/D(11)$ 
590 GOSUB 1670
600 T$="RETURN ON EQUITY"
610  $X0=(D(6)-D(7))/D(13)$ 
620 GOSUB 1670
630 T$="RETURN ON INVESTED CAPITAL"
640 X1=2
650  $X0=(D(6)-D(7))/(D(11)-D(12))$ 
660 GOSUB 1670
670 T$="EARNINGS PER SHARE"
680 X1=3
690  $X0=(D(6)-D(7))/D(14)$ 
700 GOSUB 1670
710 T$="OPERATING RATIO"
720 X1=1
730  $X0=(D(1)-D(6))/D(1)$ 
740 GOSUB 1670
750 PRINT
760 PRINT "","-----LIQUIDITY-----"
770 T$="NET WORKING CAPITAL"
780 X1=3
790 REM CALCULATE CURRENT ASSETS
800  $C1=D(8)+D(9)+D(10)+D(3)$ 
810 REM CALCULATE LONG-TERM DEBT
820  $L0=D(11)-D(12)-D(13)$ 
830  $X0=C1-D(12)$ 
840 GOSUB 1670
850 T$="ACID TEST (QUICK) RATIO"

```

```

860 X1=1
870 X0=(C1-D(3))/D(12)
880 GOSUB 1670
890 T$="CURRENT RATIO"
900 X0=C1/D(12)
910 GOSUB 1670
920 PRINT
930 PRINT "", "-----ACTIVITY-----"
940 T$="SALES PER DAY"
950 X1=3
960 X0=D(1)/365
970 GOSUB 1670
980 T$="DAYS SALES OUTSTANDING"
990 X1=0
1000 X0=D(9)/(D(1)/365)
1010 GOSUB 1670
1020 PRINT "DAYS"
1030 REM IF NO INVENTORY DATA, SKIP PRINTING
1040 IF D(2)+D(3)=0 THEN 1090
1050 T$="INVENTORY TURNOVER"
1060 X0=D(4)/((D(2)+D(3))/2)
1070 GOSUB 1670
1080 PRINT "TIMES"
1090 PRINT
1100 PRINT "", "-----INDEBTEDNESS-----"
1110 T$="CREDITORS' INTEREST IN FIRM"
1120 X1=2
1130 X0=(D(11)-D(13))/D(11)
1140 GOSUB 1670
1150 T$="TIMES INTEREST EARNED"
1160 X1=0
1170 X0=(D(6)+D(7)+D(5))/D(5)
1180 GOSUB 1670
1190 T$="DEBT TO EQUITY"
1200 X1=1
1210 X0=(D(11)-D(13))/D(13)
1220 GOSUB 1670
1230 T$="LONG-TERM DEBT TO NET WORTH"
1240 X0=L0/D(13)
1250 GOSUB 1670
1260 T$="LONG-TERM DEBT TO CAPITAL"
1270 X0=L0/(L0+D(13))
1280 GOSUB 1670
1300 PRINT "", "-----EQUITY-----"
1310 T$="STOCKHOLDERS' INTEREST IN FIRM"
1320 X1=2
1330 X0=(D(13))/D(11)
1340 GOSUB 1670
1350 T$="PAYOUT RATIO"
1360 X1=1
1370 X0=D(16)/((D(6)-D(7))/D(14))
1380 GOSUB 1670
1390 T$="EARNINGS YIELD"
1400 X1=2
1410 X0=((D(6)-D(7))/D(14))/D(15)

```



```

1420 GOSUB 1670
1430 T$="BOOK VALUE/SHARE"
1440 X1=3
1450 X0=D(13)/D(14)
1460 GOSUB 1670
1470 T$="PRICE/EARNINGS RATIO"
1480 X1=1
1490 X0=D(15)/((D(6)-D(7))/D(14))
1500 GOSUB 1670
1510 T$="DIVIDEND YIELD"
1520 X1=2
1530 X0=D(16)/D(15)
1540 GOSUB 1670
1550 PRINT
1560 PRINT "DO YOU WANT ANOTHER ANALYSIS (Y/N)"
1570 INPUT T$
1580 IF T$="Y" THEN 280
1590 IF T$<>"N" THEN 1560
1600 GOTO 1840
1610 REM DATA ENTRY ROUTINE
1620 READ T$
1630 PRINT TAB(31-LEN(T$));T$;
1640 INPUT D(I)
1650 RETURN
1660 REM SUBROUTINE TO PRINT RATIOS & TURNOVER DATA
1670 PRINT TAB(31-LEN(T$));T$;
1680 X0=INT(X0*1000+.5)/1000
1690 REM RATIO FORMAT IF X1=1
1700 IF X1=1 THEN 1780
1710 REM RATE FORMAT IF X1=2
1720 IF X1=2 THEN 1800
1730 REM DOLLAR FORMAT IF X1=3
1740 IF X1=3 THEN 1820
1750 REM DEFAULT TO NO FORMAT IF X1=0
1760 PRINT X0;
1770 RETURN
1780 PRINT X0;" : 1"
1790 RETURN
1800 PRINT X0*100;" %"
1810 RETURN
1820 PRINT " $";X0
1830 RETURN
1840 END

```

## References

- Slavin, Albert, and Reynolds, Isaac. *Basic Accounting* (3rd ed.). Hinsdale, Ill.: Dryden Press, 1975.
- Solomon, Ezra. *An Introduction to Financial Management*. Santa Monica: Goodyear Publishing Company, 1977.

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## Profit Sharing Contributions

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This program calculates the profit sharing contributions for up to 250 employees. Some profit sharing plans are not "integrated," that is, the contribution made for each employee is exactly proportionate to his salary. If his compensation is 5% of the total compensation of all participants, then he is allotted 5% of the total contribution for that year, and so on.

Integrated profit sharing plans are less straightforward. In this case, a salary level no higher than the current social security wage base (\$22,900 in 1979, \$25,900 in 1980) is chosen as the integration level. Each employee whose salary exceeds the integration level receives a percentage (not over 7%) of the amount by which his earnings exceed the integration level. The remainder of the total contribution is distributed proportionate to salary. If the integrated portion of the total contribution exceeds the total, it is reduced proportionately. If this happens, those whose salary is less than the integration level receive nothing.

This program handles both integrated and non-integrated plans of up to 250 participants. You first enter the name and salary of each employee/participant. After you enter the last employee's name and salary, enter anything for the name, and -1 for the salary when the program requests them. The program then prints out the total of the salaries, and the usual 15% limit on contributions. You then enter the amount of the contribution as a decimal fraction of the total compensation. You are asked if the plan is integrated and, if so, what the integration level and percentage are.

The program then prints a table showing each employee's name, salary, and the amount of his allocation, divided into integrated and non-integrated portions. The program prints the totals for all employees, and then allows you to go back and change some or all of the data.

### Example

The following employees are all participants in a profit sharing plan:

Name	Salary
Connell	\$150,000
Johnson	22,900
Smith	15,000
Jones	12,000
Brown	10,000

Assuming a 15% company contribution, what allocation would be made to each employee in a non-integrated plan?

Answer:

PROFIT SHARING CONTRIBUTIONS  
ENTER EACH EMPLOYEE'S NAME AND SALARY  
ENTER -1 AS THE SALARY TO END ENTRY

? CONNELL,150000

? JOHNSON,22900

? SMITH,15000

? JONES,12000

? BROWN,10000

? A,-1

TOTAL COMPENSATION= 209900

15% LIMITATION= 31485

P/S % CONTRIBUTION AS A DECIMAL=

? .15

IS PLAN INTEGRATED? (Y/N)

? N

NAME	SALARY	INTEGRATED PORTION	NON-INTEG. PORTION	TOTAL
CONNELL	150000	0	22500	22500
JOHNSON	22900	0	3435	3435
SMITH	15000	0	2250	2250
JONES	12000	0	1800	1800
BROWN	10000	0	1500	1500
TOTALS	209900	0	31485	31485

WANT DIFFERENT SALARIES? (Y/N)

? N

DIFFERENT CONTRIBUTION? (Y/N)

? N

CHANGE WHETHER INTEGRATED? (Y/N)

? N

DIFFERENT INTEGRATION LEVEL? (Y/N)

? N

DIFFERENT INTEGRATION %? (Y/N)

? N

### Practice Problems

1. For the same group of employees, what would be the allocations in a plan integrated at 3% over \$15,000?

Answer: Connell: \$23,486.40; Johnson: \$3,204.29; Smith: \$1,943.64; Jones: \$1,554.91; Brown: \$1,295.76.

2. If the plan is integrated at 7% over \$22,900, what are the allocations for these same employees?

Answer: Connell: \$25,038.97; Johnson: \$2,464.34; Smith: \$1,614.20; Jones: \$1,291.36; Brown: \$1,076.13.

## Program Listing

```
5 PRINT "PROFIT SHARING CONTRIBUTIONS"
9 REM ROUNDOFF FUNCTION
10 DEF FNR(X)=INT(100*X+.5)/100
20 DIM A$(250),B(250),C(250),D(250)
120 PRINT "ENTER EACH EMPLOYEE'S NAME AND SALARY"
140 PRINT "ENTER -1 AS THE SALARY TO END ENTRY"
150 K=0
160 J=1
170 INPUT A$(J),B(J)
180 IF B(J)=-1 THEN 240
190 K=K+B(J)
200 J=J+1
210 GOTO 170
240 J=J-1
250 PRINT "TOTAL COMPENSATION=";K
260 PRINT "15% LIMITATION=";FNR(.15*K)
270 PRINT "P/S % CONTRIBUTION AS A DECIMAL="
280 INPUT M
290 IF M>=1 THEN 270
295 IF M<=0 THEN 270
300 PRINT "IS PLAN INTEGRATED? (Y/N)"
310 INPUT Y$
320 IF Y$="N" THEN 640
330 IF Y$<>"Y" THEN 300
360 PRINT "INTEGRATION LEVEL="
370 INPUT L
390 PRINT "INTEGRATION % AS A DECIMAL="
400 INPUT P
420 S=0
430 H=0
439 REM CALCULATE INTEGRATED PORTION FOR EACH EMPLOYEE
440 FOR I=1 TO J
450 IF B(I)>L THEN 460
453 C(I)=0
456 GOTO 490
460 C(I)=FNR(P*(B(I)-L))
470 S=S+1
480 H=H+C(I)
490 NEXT I
500 IF H<M*K THEN 650
510 IF H>M*K THEN 520
512 FOR I=1 TO J
514 D(I)=0
516 NEXT I
518 GOTO 760
520 R=0
530 T=0
539 REM REDUCE INTEGRATED AMOUNT TO TOTAL CONTRIBUTION
540 FOR I=1 TO J
550 IF C(I)=0 THEN 620
560 T=T+1
570 IF T=S THEN 610
580 C(I)=FNR(C(I)*M*K/H)
590 R=R+C(I)
```

```

600 GOTO 620
610 C(I)=M*K-R
620 NEXT I
630 GOTO 760
640 H=0
642 FOR I=1 TO J
644 C(I)=0
646 NEXT I
650 G=M-H/K
669 REM CALCULATE NON-INTEGRATED PORTION
670 FOR I=1 TO J
690 D(I)=FNR(B(I)*G)
720 NEXT I
760 Q=0
770 X=0
780 PRINT,"          INTEGRATED NON-INTEG."
790 PRINT "NAME","SALARY","PORTION","PORTION","TOTAL"
799 REM PRINT OUT RESULTS
800 FOR I=1 TO J
820 X=X+C(I)+D(I)
830 Q=Q+D(I)
840 PRINT A$(I),B(I),C(I),D(I),C(I)+D(I)
850 NEXT I
855 PRINT
860 PRINT "TOTALS",K,
870 IF H>=M*K THEN 900
880 PRINT H,
890 GOTO 910
900 PRINT M*K,
910 PRINT Q,X
920 PRINT
930 PRINT "WANT DIFFERENT SALARIES? (Y/N)"
940 INPUT Z$
950 IF Z$="Y" THEN 120
960 PRINT "DIFFERENT CONTRIBUTION? (Y/N)"
970 INPUT Z$
980 IF Z$="Y" THEN 270
990 PRINT "CHANGE WHETHER INTEGRATED? (Y/N)"
1010 INPUT Z$
1020 IF Z$="Y" THEN 300
1030 PRINT "DIFFERENT INTEGRATION LEVEL? (Y/N)"
1040 INPUT Z$
1050 IF Z$="Y" THEN 360
1060 PRINT "DIFFERENT INTEGRATION %? (Y/N)"
1070 INPUT Z$
1080 IF Z$="Y" THEN 390
1090 END

```

## Reference

U.S. Internal Revenue Service Code, Sections 401-04.

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## Checkbook Reconciliation

---

This program can remove a considerable burden from you each time you reconcile your checking account. Since the computer performs all of the addition and subtraction, the chance for errors to occur is greatly reduced.

You must enter the ending balance from your statement, then each deposit or credit made since the statement date. After you have entered all outstanding deposits and credits, enter zero. This signals the program to continue to the next section, entry of outstanding checks. Enter check and other debit amounts as you did for deposits, and enter zero when all outstanding checks and debits have been entered.

You should enter only positive dollar amounts for each response. The exception is that you may enter negative amounts for your previous balance and your checkbook balance.

If your account won't balance, check all of your entries to make sure they are complete and correct. Do your check register entries match the amounts on the cancelled checks? Have you entered all checks, deposits, and automatic debits and credits? If you can't find any mistakes, call your bank.

### Example

Janet's checking account statement does not show the \$600.00 paycheck she deposited yesterday. She also wrote 2 checks that aren't shown either; one for \$87.32, and one for \$250.00. If the ending balance from the statement is \$348.55, Janet's check register shows a balance of \$614.54, and service charges on the statement are \$3.31, what is her adjusted account balance? Is Janet's account balanced?

Answer: Janet's adjusted balance is \$611.23. Her account is balanced.

### CHECKBOOK RECONCILIATION

WHAT IS THE ENDING BALANCE FROM THE STATEMENT? 348.55

ENTER THE AMOUNT OF EACH DEPOSIT NOT SHOWN ON THE STATEMENT  
(ENTER ZERO WHEN ALL OUTSTANDING DEPOSITS ARE ENTERED)

? 600

? 0

ENTER THE AMOUNT OF EACH CHECK NOT SHOWN ON THE STATEMENT  
(ENTER ZERO WHEN ALL OUTSTANDING CHECKS ARE ENTERED)

? 87.32

? 250

? 0

ACCOUNT BALANCE = \$ 611.23

ENTER YOUR CHECKBOOK BALANCE? 614.54

ENTER THE AMOUNT OF SERVICE CHARGES? 3.31

ADJUSTED ACCOUNT BALANCE = \$ 611.23

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA (Y/N)? N

## Practice Problems

1. Ending balance is \$352.13. Not shown on the statement are 3 deposits of \$100.00 each, and 6 checks: \$159.21, \$25.00, \$14.75, \$29.54, \$45.67, and \$22.50. What is the account balance? The checkbook balance is \$358.97. Service charges on this statement are \$3.51. What is the adjusted account balance? Does the account balance?

Answer: The account balance is \$355.46. The adjusted account balance is \$355.46. Yes, the account does balance.

2. Ending balance is -\$17.39. One deposit of \$250.00 is outstanding, as are 3 checks: \$50.00, \$25.00, and \$12.98. A pre-authorized withdrawal of \$35.00 has also occurred, but is not shown on this statement. What is the account balance? If the checkbook balance is \$118.99, and service charges are \$9.36, what is the adjusted account balance? Is the account balanced?

Answer: The account balance is \$109.63. The adjusted account balance is \$109.63. Yes, the account is balanced.

## Program Listing

```
10 PRINT "CHECKBOOK RECONCILIATION"
20 REM - FUNCTION TO DETERMINE IF POSITIVE DOLLAR AMOUNT WAS ENTERED
30 DEF FNB(X)=INT(X*100+.5)/100*SGN(X)
40 PRINT
50 PRINT "WHAT IS THE ENDING BALANCE FROM THE STATEMENT";
60 INPUT E
70 REM - SPECIAL TEST FOR VALID INPUT (NEGATIVE NUMBER ALLOWED)
80 IF E*100=INT(E*100) THEN 120
90 REM - INVALID AMOUNT. DISPLAY ERROR, LOOP TO RE-ENTER
100 GOSUB 680
110 GOTO 50
120 PRINT
130 PRINT "ENTER THE AMOUNT OF EACH DEPOSIT NOT SHOWN ON THE STATEMENT"
140 PRINT "(ENTER ZERO WHEN ALL OUTSTANDING DEPOSITS ARE ENTERED)"
150 D=0
160 INPUT A
170 REM - ALL DEPOSITS ENTERED?
180 IF A=0 THEN 260
190 REM - NO, TEST FOR VALID ENTRY
200 IF FNB(A)=A THEN 240
210 REM - INVALID, PRINT STANDARD ERROR, LOOP TO RE-ENTER
220 GOSUB 720
230 GOTO 160
240 D=D+A
250 GOTO 160
260 PRINT
270 PRINT "ENTER THE AMOUNT OF EACH CHECK NOT SHOWN ON THE STATEMENT"
280 PRINT "(ENTER ZERO WHEN ALL OUTSTANDING CHECKS ARE ENTERED)"
290 C=0
300 INPUT A
310 REM - ALL OUTSTANDING CHECKS ENTERED?
320 IF A=0 THEN 400
330 REM - NO, TEST FOR VALID ENTRY
340 IF FNB(A)=A THEN 380
350 REM - INVALID, PRINT STANDARD ERROR, LOOP TO RE-ENTER
360 GOSUB 720
370 GOTO 300
```

```

380 C=C+A
390 GOTO 300
400 PRINT
410 PRINT "ACCOUNT BALANCE = $";(E+D)-C
420 PRINT
430 PRINT "ENTER YOUR CHECKBOOK BALANCE";
440 INPUT B
450 PRINT "ENTER THE AMOUNT OF SERVICE CHARGES";
460 INPUT S
470 REM - TEST FOR VALID ENTRY
480 IF FNB(S)=S THEN 520
490 REM - INVALID, PRINT STANDARD ERROR, LOOP TO RE-ENTER
500 GOSUB 720
510 GOTO 450
520 PRINT
530 PRINT "ADJUSTED ACCOUNT BALANCE = $";B-S
540 IF (E+D-C)=(B-S) THEN 620
550 PRINT
560 PRINT "YOUR ACCOUNT IS OUT OF BALANCE."
570 PRINT "MAKE SURE YOU HAVE INCLUDED ALL TRANSACTIONS AGAINST"
580 PRINT "THIS ACCOUNT, INCLUDING AUTOMATIC DEPOSITS AND"
590 PRINT "INTEREST PAYMENTS, AS WELL AS PRE-AUTHORIZED WITHDRAWALS."
600 PRINT
610 REM - END PROGRAM OR RESTART?
620 PRINT
630 PRINT "WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA (Y/N)";
640 INPUT Z$
650 IF Z$="Y" THEN 40
660 IF Z$="N" THEN 760
670 GOTO 630
680 PRINT
690 PRINT "ERROR: ENTER A VALID DOLLAR AMOUNT ONLY."
700 PRINT
710 RETURN
720 PRINT
730 PRINT "ERROR: ENTER A POSITIVE VALID DOLLAR AMOUNT ONLY."
740 PRINT
750 RETURN
760 END

```



---

## Home Budgeting

---

This program sets up a cash budget for personal use, allowing for a variety of expenses which can occur at many different times. Once you enter the income and expense information which the program requests, day-by-day details of income and expenses print as they occur. The program also allows you to use credit cards as a means of paying expenses when the cash you have is insufficient to meet your obligations. Or, if you wish, you can delay them until the next time they come up.

To use the program, enter the date where the budget will begin. The program then guides you through a series of entries, starting with net income(s), followed by secured loans, credit cards and, finally, normal living expenses. If your budget does not include items which the program asks for, just enter zero (0) for those items. The program will then skip to the next budget item.

Whenever you have a budget item to enter, you will have to enter its periodic amount, how often it occurs, and when it will occur next. The exceptions to this are secured loans and credit cards, which ask for more information. The Periodic amount is the amount you regularly receive as income, or pay as an expense. When you enter how often the budget item occurs, it must be an integer from 1 to 99, inclusive. This number tells the program how many times per year the item occurs (1=yearly, 2=semi-annually, 4=quarterly, 6=bimonthly, 12=monthly, 24=semimonthly, 26=biweekly, and 52=weekly).

If the next date for the budget item happens to be the same as the budget start date, enter zero (0). Otherwise, enter the next date as one number (e.g., 91580=Sept. 15, 1980). You can enter a date months or even years after the budget start date if you like. When the program performs its cashflow analysis, it will "activate" future income or expenses when it reaches the date you specify.

With secured loans, you have to enter the remaining balance of the loan as well as the periodic amount, frequency and next date. When you enter credit card information, you will input the annual percentage rate for the card, the remaining balance and its authorized credit limit. The program automatically calculates the number and amount of remaining payments for each credit card, and displays them. If you want to change the payment which the program calculates, just specify a new periodic payment of a higher or lower amount. Note: the program will calculate an even stream of payments to make budgeting more predictable. When the remaining balance of the credit card goes below the calculated payment amount during the cashflow analysis, only the remaining balance is paid.

Once you have entered all of the budget items, the program will ask how much cash you have on hand. Enter this amount, and the program will begin its cashflow analysis. At the end of each month's detail, total cash inflows and outflows print. At this point, you can choose to go on to the next month's analysis or stop the program.

Because you will be entering a significant amount of data in order to run this program, you should know how to correct data entry errors. You can only correct errors which you make on the current budget item; that is, you cannot backtrack to the fifth item when you are on the tenth.

On a current budget item, you can move as far back as the periodic amount entry by entering -1. For example, you notice that you have entered the wrong periodic amount for salary 1, and the program now wants you to enter the next date for this item. Rather than entering the next date for salary 1, enter -1. The computer will accept this entry and then ask you to enter the periodic amount for salary 1 again.

### Program Notes

Home budgeting/cashflow allows for a maximum of 3 incomes, 3 loans, 5 credit cards and 25 expense items. At present, the program will allocate cash to loans first, then credit cards, and finally other expenses. The expenses are arranged in descending order of importance; i.e., if a loan, charge card and restaurant expense all appear on the same day, the program will allocate cash to the loan first and to the restaurant expense last.

Changing this program to allow for more budget items is a three-step process. First, change line 20, substituting the terms A, B, C, and D in parentheses with actual numbers. These terms are explained below.

20 DIM D(12), IO(A,2), CO(B,3), C1(C,5), C1\$(C), EO(D,2)

A = Maximum number of incomes  
 B = Maximum number of secured loans  
 C = Maximum number of charge cards  
 D = Maximum number of expense items

The second step is to put descriptions of the extra budget items in the DATA statements at the beginning of the program. You can add any extra loans by placing DATA statements between lines 90 and 100 which contain descriptions of the loans. Note: you do not need to change DATA statements to allow for more incomes or more credit cards. To add more expenses, add DATA statements anywhere from lines 110 through 180.

The third and last step is to change FOR/NEXT loops in the program. If you change the number of secured loans, be sure to also change lines 530 and 1080 of the program. Currently they are set for 3 iterations. Change the number 3 in these two statements to the new number of secured loans. If you have added or eliminated expense items, you will need to change lines 750 and 1360. Change the number 25 in these two statements to the new number of expense items.

### Example

#### HOME BUDGETING/CASHFLOW MODEL

<pre> DATE TO START ANALYSIS FROM: ENTER MONTH-DAY-YEAR (MMDDYY)? 90180 -----NET SALARY 1 ----- PERIODIC AMOUNT FOR INCOME? 512 HOW MANY TIMES PER YEAR? 26 ENTER MONTH-DAY-YEAR (MMDDYY)? 90580 -----NET SALARY 2 ----- PERIODIC AMOUNT FOR INCOME? 100 HOW MANY TIMES PER YEAR? 4 ENTER MONTH-DAY-YEAR (MMDDYY)? 100180 -----NET SALARY 3 ----- PERIODIC AMOUNT FOR INCOME? 0 PERIODIC AMOUNT FOR MORTGAGE? 0 PERIODIC AMOUNT FOR CAR LOAN? 80 HOW MANY TIMES PER YEAR? 12 ENTER MONTH-DAY-YEAR (MMDDYY)?-1 CURRENT BALANCE?-1 PERIODIC AMOUNT FOR CAR LOAN?-1 PERIODIC AMOUNT FOR CAR LOAN? 95 HOW MANY TIMES PER YEAR? 12 ENTER MONTH-DAY-YEAR (MMDDYY)? 91580 CURRENT BALANCE? 1290 PERIODIC AMOUNT FOR OTHER LOAN? 0 </pre>	<p>Start analysis on Sept. 1, 1980.</p> <p>First net income is \$512.00, paid bi-weekly. The next paycheck will be on Sept. 5, 1980.</p> <p>Finish entering income data.</p> <p>Car loan payment was incorrect. -1 entry used to back up to the incorrect entry.</p>
---	--

NAME OF CREDIT CARD 1 (RETURN TO END)? VISA  
ANNUAL INTEREST RATE? 18  
CURRENT BALANCE? 525  
CREDIT LIMIT? 1000

Enter credit card 1.  
Note: calculation of payments  
allows for interest over 12 payments.

12 PAYMENTS OF \$ 52.5 NEEDED TO PAY DEBT

CHANGE AMOUNT (Y/N)? Y

ENTER DESIRED PAYMENT AMOUNT? 35

18 PAYMENTS OF \$ 35 NEEDED TO PAY DEBT

CHANGE AMOUNT (Y/N)? N

Payment was changed to a lower  
amount.

ENTER NEXT VISA BILLING DATE:

ENTER MONTH-DAY-YEAR (MMDDYY)? 92080

NAME OF CREDIT CARD 2 (RETURN TO END)? MASTERCARGE

ANNUAL INTEREST RATE? 18

CURRENT BALANCE? 230

CREDIT LIMIT? 500

12 PAYMENTS OF \$ 23 NEEDED TO PAY DEBT

CHANGE AMOUNT (Y/N)? N

ENTER NEXT MASTERCARGE BILLING DATE:

ENTER MONTH-DAY-YEAR (MMDDYY)? 92480

NAME OF CREDIT CARD 3 (RETURN TO END)?

Finish entering credit card data.

PERIODIC AMOUNT FOR PROPERTY TAX? 0

PERIODIC AMOUNT FOR RENT? 300

HOW MANY TIMES PER YEAR? 12

ENTER MONTH-DAY-YEAR (MMDDYY)? 90180

PERIODIC AMOUNT FOR LIFE INSURANCE? 12.5

HOW MANY TIMES PER YEAR? 12

ENTER MONTH-DAY-YEAR (MMDDYY)? 92480

PERIODIC AMOUNT FOR HOUSE INSURANCE? 0

PERIODIC AMOUNT FOR CAR INSURANCE? 125

HOW MANY TIMES PER YEAR? 4

ENTER MONTH-DAY-YEAR (MMDDYY)? 102180

Quarterly expense.

PERIODIC AMOUNT FOR TELEPHONE? 35

HOW MANY TIMES PER YEAR? 12

ENTER MONTH-DAY-YEAR (MMDDYY)? 90880

PERIODIC AMOUNT FOR GAS & ELECTRIC? 17

HOW MANY TIMES PER YEAR? 12

ENTER MONTH-DAY-YEAR (MMDDYY)? 91880

PERIODIC AMOUNT FOR WATER? 0

PERIODIC AMOUNT FOR TRASH PICKUP? 0

PERIODIC AMOUNT FOR GROCERIES? 25

HOW MANY TIMES PER YEAR? 52

ENTER MONTH-DAY-YEAR (MMDDYY)? 90580

Weekly expense.

PERIODIC AMOUNT FOR CLOTHING? 40

HOW MANY TIMES PER YEAR? 4

ENTER MONTH-DAY-YEAR (MMDDYY)? 110180

PERIODIC AMOUNT FOR PHYSICIAN? 30  
 HOW MANY TIMES PER YEAR? 4  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 120180

PERIODIC AMOUNT FOR DENTIST? 0

PERIODIC AMOUNT FOR DRUGS? 0

PERIODIC AMOUNT FOR TUITION? 0

PERIODIC AMOUNT FOR CHILD CARE? 0

PERIODIC AMOUNT FOR GAS/OIL? 15  
 HOW MANY TIMES PER YEAR? 52  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 90680

PERIODIC AMOUNT FOR AUTO REPAIR? 40  
 HOW MANY TIMES PER YEAR? 3  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 10181

Expense occurs every 4 months.

PERIODIC AMOUNT FOR COMMUTING? 0

PERIODIC AMOUNT FOR MEDICAL PLAN? 0

PERIODIC AMOUNT FOR HOME REPAIR? 0

PERIODIC AMOUNT FOR RESTAURANTS? 15  
 HOW MANY TIMES PER YEAR? 52  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 0

Next date for this item is the same as the budget start date.

PERIODIC AMOUNT FOR MOVIES/CONCERTS? 10  
 HOW MANY TIMES PER YEAR? 26  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 0

PERIODIC AMOUNT FOR SUBSCRIPTIONS? 0

PERIODIC AMOUNT FOR MISCELLANEOUS? 18

HOW MANY TIMES PER YEAR? 52  
 ENTER MONTH-DAY-YEAR (MMDDYY)? 91580  
 ENTER CASH ON HAND? 400

Cash available at start of analysis.

CASH FLOWS FOR 9 / 80		
OPENING CASH BALANCE \$ 400		
1	RENT	-300
1	RESTAURANTS	-15
1	MOVIES/CONCERTS	-10
FRI 5	INCOME 1	512
FRI 5	GROCERIES	-25
SAT 6	GAS/OIL	-15
MON 8	TELEPHONE	-35
MON 8	RESTAURANTS	-15
FRI 12	GROCERIES	-25
SAT 13	GAS/OIL	-15
MON 15	CAR LOAN PAYMENT	-95

MON 15	RESTAURANTS		-15
MON 15	MOVIES/CONCERTS		-10
MON 15	MISCELLANEOUS		-18
THU 18	GAS & ELECTRIC		-17
FRI 19	INCOME 1	512	
FRI 19	GROCERIES		-25
SAT 20	VISA		-35
SAT 20	GAS/OIL		-15
MON 22	RESTAURANTS		-15
MON 22	MISCELLANEOUS		-18
WED 24	MASTERCARGE		-23
WED 24	LIFE INSURANCE		-12.5
FRI 26	GROCERIES		-25
SAT 27	GAS/OIL		-15
MON 29	RESTAURANTS		-15
MON 29	MOVIES/CONCERTS		-10
MON 29	MISCELLANEOUS		-18
CASH IN: 1024		CASH OUT 836.5	Total monthly cash income and expenses.

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 10 / 80

OPENING CASH BALANCE \$ 587.5

WED 1	INCOME 2	100	
WED 1	RENT		-300
THU 2	INCOME 1	512	
THU 2	GROCERIES		-25
FRI 3	GAS/OIL		-15
SUN 5	RESTAURANTS		-15
SUN 5	MISCELLANEOUS		-18
WED 8	TELEPHONE		-35
THU 9	GROCERIES		-25
FRI 10	GAS/OIL		-15
SUN 12	RESTAURANTS		-15
SUN 12	MOVIES/CONCERTS		-10
SUN 12	MISCELLANEOUS		-18
WED 15	CAR LOAN PAYMENT		-95
THU 16	INCOME 1	512	
THU 16	GROCERIES		-25
FRI 17	GAS/OIL		-15
SAT 18	GAS & ELECTRIC		-17
SUN 19	RESTAURANTS		-15
SUN 19	MISCELLANEOUS		-18
MON 20	VISA		-35
TUE 21	CAR INSURANCE		-125
THU 23	GROCERIES		-25
FRI 24	MASTERCARGE		-23
FRI 24	LIFE INSURANCE		-12.5
FRI 24	GAS/OIL		-15
SUN 26	RESTAURANTS		-15
SUN 26	MOVIES/CONCERTS		-10
SUN 26	MISCELLANEOUS		-18
THU 30	INCOME 1	512	
THU 30	GROCERIES		-25
FRI 31	GAS/OIL		-15
CASH IN: 1636		CASH OUT 994.5	

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 11 / 80

OPENING CASH BALANCE \$ 1229

SAT 1	RENT		-300
SAT 1	CLOTHING		-40
SUN 2	RESTAURANTS		-15
SUN 2	MISCELLANEOUS		-18
THU 6	GROCERIES		-25
FRI 7	GAS/OIL		-15
SAT 8	TELEPHONE		-35
SUN 9	RESTAURANTS		-15
SUN 9	MOVIES/CONCERTS		-10
SUN 9	MISCELLANEOUS		-18
THU 13	INCOME 1	512	
THU 13	GROCERIES		-25
FRI 14	GAS/OIL		-15
SAT 15	CAR LOAN PAYMENT		-95
SUN 16	RESTAURANTS		-15
SUN 16	MISCELLANEOUS		-18
TUE 18	GAS & ELECTRIC		-17
THU 20	VISA		-35
THU 20	GROCERIES		-25
FRI 21	GAS/OIL		-15
SUN 23	RESTAURANTS		-15
SUN 23	MOVIES/CONCERTS		-10
SUN 23	MISCELLANEOUS		-18
MON 24	MASTERCARGE		-23
MON 24	LIFE INSURANCE		-12.5
THU 27	INCOME 1	512	
THU 27	GROCERIES		-25
FRI 28	GAS/OIL		-15
SUN 30	RESTAURANTS		-15
SUN 30	MISCELLANEOUS		-18

CASH IN: 1024

CASH OUT 902.5

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 12 / 80

OPENING CASH BALANCE \$ 1350.5

MON 1	RENT		-300
MON 1	PHYSICIAN		-30
THU 4	GROCERIES		-25
FRI 5	GAS/OIL		-15
SUN 7	RESTAURANTS		-15
SUN 7	MOVIES/CONCERTS		-10
SUN 7	MISCELLANEOUS		-18
MON 8	TELEPHONE		-35
THU 11	INCOME 1	512	
THU 11	GROCERIES		-25
FRI 12	GAS/OIL		-15
SUN 14	RESTAURANTS		-15
SUN 14	MISCELLANEOUS		-18
MON 15	CAR LOAN PAYMENT		-95
THU 18	GAS & ELECTRIC		-17
THU 18	GROCERIES		-25
FRI 19	GAS/OIL		-15

SAT 20	VISA		-35
SUN 21	RESTAURANTS		-15
SUN 21	MOVIES/CONCERTS		-10
SUN 21	MISCELLANEOUS		-18
WED 24	MASTERCARGE		-23
WED 24	LIFE INSURANCE		-12.5
THU 25	INCOME 1	512	
THU 25	GROCERIES		-25
FRI 26	GAS/OIL		-15
SUN 28	RESTAURANTS		-15
SUN 28	MISCELLANEOUS		-18
CASH IN: 1024			CASH OUT 859.5

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 1 / 81

OPENING CASH BALANCE \$ 1515

THU 1	INCOME 2	100	
THU 1	RENT		-300
THU 1	GROCERIES		-25
THU 1	AUTO REPAIR		-40
FRI 2	GAS/OIL		-15
SUN 4	RESTAURANTS		-15
SUN 4	MOVIES/CONCERTS		-10
SUN 4	MISCELLANEOUS		-18
THU 8	INCOME 1	512	
THU 8	TELEPHONE		-35
THU 8	GROCERIES		-25
FRI 9	GAS/OIL		-15
SUN 11	RESTAURANTS		-15
SUN 11	MISCELLANEOUS		-18
THU 15	CAR LOAN PAYMENT		-95
THU 15	GROCERIES		-25
FRI 16	GAS/OIL		-15
SUN 18	GAS & ELECTRIC		-17
SUN 18	RESTAURANTS		-15
SUN 18	MOVIES/CONCERTS		-10
SUN 18	MISCELLANEOUS		-18
TUE 20	VISA		-35
WED 21	CAR INSURANCE		-125
THU 22	INCOME 1	512	
THU 22	GROCERIES		-25
FRI 23	GAS/OIL		-15
SAT 24	MASTERCARGE		-23
SAT 24	LIFE INSURANCE		-12.5
SUN 25	RESTAURANTS		-15
SUN 25	MISCELLANEOUS		-18
THU 29	GROCERIES		-25
FRI 30	GAS/OIL		-15
CASH IN: 1124			CASH OUT 1034.5

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 2 / 81

OPENING CASH BALANCE \$ 1604.5

SUN 1	RENT		-300
SUN 1	CLOTHING		-40
SUN 1	RESTAURANTS		-15



SUN 1	MOVIES/CONCERTS		-10
SUN 1	MISCELLANEOUS		-18
THU 5	INCOME 1	512	
THU 5	GROCERIES		-25
FRI 6	GAS/OIL		-15
SUN 8	TELEPHONE		-35
SUN 8	RESTAURANTS		-15
SUN 8	MISCELLANEOUS		-18
THU 12	GROCERIES		-25
FRI 13	GAS/OIL		-15
SUN 15	CAR LOAN PAYMENT		-95
SUN 15	RESTAURANTS		-15
SUN 15	MOVIES/CONCERTS		-10
SUN 15	MISCELLANEOUS		-18
WED 18	GAS & ELECTRIC		-17
THU 19	INCOME 1	512	
THU 19	GROCERIES		-25
FRI 20	VISA		-35
FRI 20	GAS/OIL		-15
SUN 22	RESTAURANTS		-15
SUN 22	MISCELLANEOUS		-18
TUE 24	MASTERCARGE		-23
TUE 24	LIFE INSURANCE		-12.5
THU 26	GROCERIES		-25
FRI 27	GAS/OIL		-15
CASH IN: 1024		CASH OUT	869.5

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 3 / 81

OPENING CASH BALANCE \$ 1759

SUN 1	RENT		-300
SUN 1	PHYSICIAN		-30
SUN 1	RESTAURANTS		-15
SUN 1	MOVIES/CONCERTS		-10
SUN 1	MISCELLANEOUS		-18
THU 5	INCOME 1	512	
THU 5	GROCERIES		-25
FRI 6	GAS/OIL		-15
SUN 8	TELEPHONE		-35
SUN 8	RESTAURANTS		-15
SUN 8	MISCELLANEOUS		-18
THU 12	GROCERIES		-25
FRI 13	GAS/OIL		-15
SUN 15	CAR LOAN PAYMENT		-95
SUN 15	RESTAURANTS		-15
SUN 15	MOVIES/CONCERTS		-10
SUN 15	MISCELLANEOUS		-18
WED 18	GAS & ELECTRIC		-17
THU 19	INCOME 1	512	
THU 19	GROCERIES		-25
FRI 20	VISA		-35
FRI 20	GAS/OIL		-15
SUN 22	RESTAURANTS		-15
SUN 22	MISCELLANEOUS		-18
TUE 24	MASTERCARGE		-23
TUE 24	LIFE INSURANCE		-12.5



THU 26	GROCERIES	-25
FRI 27	GAS/OIL	-15
SUN 29	RESTAURANTS	-15
SUN 29	MOVIES/CONCERTS	-10
SUN 29	MISCELLANEOUS	-18
CASH IN: 1024		CASH OUT 902.5

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? Y

CASH FLOWS FOR 4 / 81

OPENING CASH BALANCE \$ 1880.5

WED 1	INCOME 2	100	
WED 1	RENT		-300
THU 2	INCOME 1	512	
THU 2	GROCERIES		-25
FRI 3	GAS/OIL		-15
SUN 5	RESTAURANTS		-15
SUN 5	MISCELLANEOUS		-18
WED 8	TELEPHONE		-35
THU 9	GROCERIES		-25
FRI 10	GAS/OIL		-15
SUN 12	RESTAURANTS		-15
SUN 12	MOVIES/CONCERTS		-10
SUN 12	MISCELLANEOUS		-18
WED 15	CAR LOAN PAYMENT		-95
THU 16	INCOME 1	512	
THU 16	GROCERIES		-25
FRI 17	GAS/OIL		-15
SAT 18	GAS & ELECTRIC		-17
SUN 19	RESTAURANTS		-15
SUN 19	MISCELLANEOUS		-18
MON 20	VISA		-35
TUE 21	CAR INSURANCE		-125
THU 23	GROCERIES		-25
FRI 24	MASTERCARD		-23
FRI 24	LIFE INSURANCE		-12.5
FRI 24	GAS/OIL		-15
SUN 26	RESTAURANTS		-15
SUN 26	MOVIES/CONCERTS		-10
SUN 26	MISCELLANEOUS		-18
THU 30	INCOME 1	512	
THU 30	GROCERIES		-25
CASH IN: 1636		CASH OUT 979.5	

DO YOU WANT TO SEE THE NEXT MONTH (Y/N)? N

### Program Listing

```

10 REM HOME BUDGETING/CASHFLOW ANALYSIS 4/9/80 11:00 AM
20 DIM D(12),IO(3,2),CO(4,3),C1(5,5),C1$(5),EO(25,2)
30 REM D() --DAY OFFSET FACTORS
40 REM IO() --SALARIED INCOME
50 REM C1() --CREDIT INSTRUMENTS
60 REM EO() --EXPENSES
70 REM C1$() --DESCRIPTIONS OF CREDIT CARDS

```

```

80 REM C0() --FIXED-TERM LOANS
90 DATA "MORTGAGE","CAR LOAN","OTHER LOAN", FURNITURE
100 REM EXPENSES
110 DATA "PROPERTY TAX","RENT"
120 DATA "LIFE INSURANCE","HOUSE INSURANCE","CAR INSURANCE"
130 DATA "TELEPHONE","GAS & ELECTRIC","WATER","TRASH PICKUP"
140 DATA "GROCERIES","CLOTHING","PHYSICIAN","DENTIST"
150 DATA "DRUGS","TUITION","CHILD CARE","GAS/OIL"
160 DATA "AUTO REPAIR","COMMUTING","MEDICAL PLAN"
170 DATA "HOME REPAIR","RESTAURANTS","MOVIES/CONCERTS"
180 DATA "SUBSCRIPTIONS","MISCELLANEOUS"
190 D(1)=31 J
200 D(2)=28 F
210 D(3)=31 W
220 D(4)=30 A
230 D(5)=31 W
240 D(6)=30 J
250 D(7)=31 J
260 D(8)=31 A
270 D(9)=31 S - 30
280 D(10)=31 O
290 D(11)=30 N
300 D(12)=31 D
310 D$="SAT SUN MON TUE WED THU FRI"
320 PRINT "HOME BUDGETING/CASHFLOW MODEL"
330 PRINT
340 PRINT "      DATE TO START ANALYSIS FROM:"
350 GOSUB 2990
360 D1=D2
370 Y1=Y
380 M1=M
390 D4=Y*10000+M*100+D2
400 PRINT
410 REM ENTER INCOMES--AMOUNTS & FREQUENCY
420 I2=0
430 X$="INCOME"
440 PRINT "-----NET SALARY";I2+1;"-----"
450 GOSUB 2360
460 IF A2(1)=0 THEN 510
470 I2=I2+1
480 I0(I2,1)=A2(1)
490 I0(I2,2)=A2(2)
500 GOTO 440
510 PRINT
520 REM ENTER SECURED LOANS
530 FOR I=1 TO 3
540 READ X$
550 PRINT
560 GOSUB 2360
570 IF A2(1)=0 THEN 640
580 IF A2(1)<0 THEN 550
590 C0(I,1)=A2(1)
600 C0(I,2)=A2(2)
610 PRINT "      CURRENT BALANCE";
620 INPUT C0(I,3)
630 IF C0(I,3)<1 THEN 560

```

J 31  
F 31  
W 31  
A 31

if 1272 then 510.

```

640 NEXT I
650 REM ENTER CREDIT CARDS AND DESCRIPTIONS
660 REM MONTHLY PAYMENTS ARE ASSUMED
670 PRINT
680 K=1
690 GOSUB 2020
700 IF C1$(K)<=" " THEN 730
710 K=K+1
720 GOTO 690
730 C4=K
740 REM ENTER EXPENSES
750 FOR K=1 TO 25
760 PRINT
770 READ X$
780 GOSUB 2360
790 E0(K,1)=A2(1)
800 E0(K,2)=A2(2)
810 NEXT K
820 REM INPUT PRESENT CASH RESERVES
830 PRINT "ENTER CASH ON HAND";
840 INPUT B0
850 REM BEGIN ANALYSIS
860 PRINT
870 PRINT "CASH FLOWS FOR ";M1;"/";Y1
880 PRINT "OPENING CASH BALANCE $";B0
890 E1=0
900 I1=0
910 FOR K1=D1 TO D(M1)
920 RESTORE
930 FOR J=1 TO (I2)
940 REM CHECK FOR INCOME
950 IF INT(I0(J,2))>D4 THEN 1060
960 B0=B0+I0(J,1)
970 I1=I1+I0(J,1)
980 M=M1
990 D2=D1
1000 Y=Y1
1010 D3=(I0(J,2)-INT(I0(J,2)))*100+.5
1020 A2(1)=D3/100
1030 GOSUB 2510
1040 I0(J,2)=A2(1)+Y*10000+M*100+D2
1050 PRINT A$;D1;TAB(9);"INCOME";J;TAB(35);I0(J,1)
1060 NEXT J
1070 REM CALCULATE OUTFLOWS FOR FIXED-TERM LOANS
1080 FOR J=1 TO 3
1090 READ X$
1100 IF C0(J,3)=0 THEN 1200
1110 IF INT(C0(J,2))>D4 THEN 1200
1120 IF C0(J,3)>C0(J,1) THEN 1140
1130 C0(J,1)=C0(J,3)
1140 A2(1)=C0(J,1)
1150 A2(2)=C0(J,2)
1160 GOSUB 1700
1170 C0(J,2)=(C0(J,2)-INT(C0(J,2)))+Y*10000+M*100+D2
1180 PRINT A$;D1;TAB(9);X$;" PAYMENT";TAB(50);-1*A2(1)

```

```

1190 C0(J,3)=C0(J,3)-A2(1)
1200 NEXT J
1210 REM CALCULATE OUTFLOWS FOR CHARGE CARDS
1220 FOR J=1 TO C4
1230 IF C1(J,5)>D4 THEN 1340
1240 IF C1(J,2)=0 THEN 1340
1250 IF C1(J,2)>C1(J,4) THEN 1270
1260 C1(J,4)=C1(J,2)
1270 A2(1)=C1(J,4)
1280 A2(2)=C1(J,5)+.12
1290 X$=C1$(J)
1300 GOSUB 1700
1310 PRINT A$;D1;TAB(9);C1$(J);TAB(50);-1*A2(1)
1320 C1(J,2)=C1(J,2)-A2(1)
1330 C1(J,5)=Y*10000+M*100+D2
1340 NEXT J
1350 REM CALCULATE OUTFLOWS FOR EXPENSES
1360 FOR J=1 TO 25
1370 READ X$
1380 IF E0(J,1)=0 THEN 1450
1390 IF INT(E0(J,2))>D4 THEN 1450
1400 A2(1)=E0(J,1)
1410 A2(2)=E0(J,2)
1420 GOSUB 1700
1430 PRINT A$;D1;TAB(9);X$;TAB(50);-1*A2(1)
1440 E0(J,2)=(E0(J,2)-INT(E0(J,2)))+Y*10000+M*100+D2
1450 NEXT J
1460 D1=D1+1
1470 D4=Y1*10000+M1*100+D1
1480 M=M1
1490 D2=D1
1500 Y=Y1
1510 GOSUB 2890
1520 NEXT K1
1530 D3=1
1540 D2=D(M1)
1550 M=M1
1560 Y=Y1
1570 GOSUB 2750
1580 D1=1
1590 M1=M
1600 Y1=Y
1610 GOSUB 2890
1620 D4=Y1*10000+M1*100+D1
1630 PRINT TAB(22);"CASH IN: ";I1;TAB(41);"CASH OUT";E1
1640 PRINT
1650 PRINT "DO YOU WANT TO SEE THE NEXT MONTH (Y/N)";
1660 INPUT X0$
1670 IF X0$="Y" THEN 870
1680 IF X0$="N" THEN 3310
1690 GOTO 870
1700 REM APPLY EXPENSES
1710 X0$=""
1720 IF B0-A2(1)>=0 THEN 1910
1730 PRINT "CASH NEEDED FOR: ";X$;" $";A2(1);"ON HAND: ";B0

```

```

1740 PRINT "ENTER D=DELAY EXPENSE; C=USE CREDIT CARD";
1750 INPUT X0$
1760 IF X0$="D" THEN 1980
1770 IF X0$<>"C" THEN 1740
1780 PRINT "CREDIT CARD NUMBER (1-";C4;"OR ZERO)";
1790 INPUT X0
1800 IF X0<1 THEN 1740
1810 IF X0>C4 THEN 1780
1820 IF C1(X0,2)+A2(1)<=C1(X0,3) THEN 1850
1830 PRINT "AVAILABLE ";C1$(X0); " CREDIT:$";C1(X0,3)-C1(X0,2)
1840 GOTO 1780
1850 C1(X0,2)=C1(X0,2)+A2(1)
1860 K=X0
1870 X0$="1"
1880 GOSUB 2160
1890 X0$=""
1900 RETURN
1910 B0=B0-A2(1)
1920 E1=E1+A2(1)
1930 D3=(A2(2)-INT(A2(2)))*100
1940 Y=INT(A2(2)/10000)
1950 M=INT((A2(2)-Y*10000)/100)
1960 D2=INT((A2(2)-(Y*10000+M*100)))
1970 REM CALCULATE NEXT DATE
1980 GOSUB 2510
1990 IF X0$<>"D" THEN 2010
2000 PRINT "EXPENSE IS DELAYED UNTIL";M;"/";D2;"/";Y
2010 RETURN
2020 REM ROUTINE TO ENTER CREDIT & CHARGE CARD DATA
2030 PRINT "NAME OF CREDIT CARD";K;"(RETURN TO END)";
2040 INPUT C1$(K)
2050 IF C1$(K)<=" " THEN 2350
2060 PRINT "                ANNUAL INTEREST RATE";
2070 INPUT C1(K,1)
2080 IF C1(K,1)<0 THEN 2020
2090 PRINT "                CURRENT BALANCE";
2100 INPUT C1(K,2)
2110 IF C1(K,2)<0 THEN 2060
2120 PRINT "                CREDIT LIMIT";
2130 INPUT C1(K,3)
2140 IF C1(K,3)<0 THEN 2090
2150 IF C1(K,1)=0 THEN 2290
2160 C1(K,4)=INT(.1*C1(K,2)*100+.5)/100
2170 I1=C1(K,1)/100
2180 P1=C1(K,2)
2190 A1=C1(K,4)
2200 IF P1<=0 THEN 2290
2210 GOSUB 3260
2220 PRINT A1;"PAYMENTS OF $";C1(K,4);"NEEDED TO PAY DEBT"
2230 PRINT "                CHANGE AMOUNT (Y/N)";
2240 INPUT X1$
2250 IF X1$<>"Y" THEN 2290
2260 PRINT "                ENTER DESIRED PAYMENT AMOUNT";
2270 INPUT C1(K,4)
2280 GOTO 2180

```

```

2290 IF X0$="1" THEN 2350
2300 PRINT "ENTER NEXT ";C1$(K);" BILLING DATE:"
2310 A2(2)=0
2320 GOSUB 2470
2330 IF X1=-1 THEN 2120
2340 C1(K,5)=A2(2)
2350 RETURN
2360 REM ROUTINE TO CALCULATE EXPENSE FREQUENCIES
2370 REM A2() ARRAY CONTAINS RESULTS
2380 PRINT TAB(16-LEN(X$));"PERIODIC AMOUNT FOR ";X$;
2390 INPUT A2(1)
2400 IF A2(1)<=0 THEN 2500—
2410 PRINT "          HOW MANY TIMES PER YEAR";
2420 INPUT A2(2)
2430 IF A2(2)<=0 THEN 2380
2440 IF A2(2)<100 THEN 2470
2450 PRINT "FREQUENCY CANNOT EXCEED 99 DAYS"
2460 GOTO 2410
2470 GOSUB 2990
2480 IF X1=-1 THEN 2500
2490 A2(2)=A2(2)/100+Y*10000+M*100+D2
2500 RETURN
2510 REM FIND NEXT MONTHLY,BIMONTHLY OR QUARTERLY OCCURRENCE
2520 IF 24/D3<>INT(24/D3) THEN 2740
2530 IF D3=24 THEN 2620
2540 FOR K=1 TO 12/D3
2550 M=M+1
2560 IF M<=12 THEN 2590
2570 M=1
2580 Y=Y+1
2590 NEXT K
2600 RETURN
2610 REM CALCULATE NEXT SEMIMONTHLY OCCURRENCE
2620 IF D2<>D(M) THEN 2650
2625 IF D2<1 THEN 2650
2630 D2=15
2640 GOTO 2690
2650 IF D2>D(M) THEN 2680
2660 D2=D2+15
2670 RETURN
2680 D2=D2-15
2690 M=M+1
2700 IF M<=12 THEN 2730
2710 Y=Y+1
2720 M=1
2730 RETURN
2740 D3=INT(365.25/D3)
2750 REM CALCULATE A DAY D3 DAYS FROM M/D2/Y
2760 IF D2+D3<=D(M) THEN 2870
2770 D3=D3-(D(M)-D2)
2780 D2=0
2790 M=M+1
2800 IF M<=12 THEN 2760
2810 Y=Y+1
2820 M=1
2830 D(2)=28

```

```

2840 IF Y/4 <> INT(Y/4) THEN 2860
2850 D(2)=29
2860 GOTO 2760
2870 D2=D2+D3
2880 RETURN
2890 REM SUBROUTINE TO CALCULATE DAY OF WEEK
2900 IF Y>1900 THEN 2920
2910 Y=Y+1900
2920 IF M>2 THEN 2950
2930 M=M+12
2940 Y=Y-1
2950 N=D2+2*M+INT(.6*(M+1))+Y+INT(Y/4)-INT(Y/100)+INT(Y/400)+2
2960 N=INT((N/7-INT(N/7))*7+.5)
2970 A$=MID$(D$, (N*3)+1,3)
2980 RETURN
2990 REM ROUTINE TO ENTER DATE
3000 REM DATE IS PASSED BACK IN M,D2 AND Y;
3010 D(2)=28
3020 PRINT "      ENTER MONTH-DAY-YEAR (MMDDYY)";
3030 INPUT X1
3040 IF X1=0 THEN 3160
3050 IF X1=-1 THEN 3190
3060 M=INT(X1/1E4)
3070 IF M>12 THEN 3020
3080 IF M<1 THEN 3020
3090 Y=(X1/100-INT(X1/100))*100
3100 IF Y/4 <> INT(Y/4) THEN 3120
3110 D(2)=29
3120 D2=INT((X1-(M*1E4+Y))/100)
3130 IF D2<1 THEN 3020
3140 IF D2>D(M) THEN 3020
3150 GOTO 3190
3160 M=M1
3170 D2=D1
3180 Y=Y1
3190 RETURN
3200 REM SUBROUTINE TO CALCULATE EVEN PAYMENTS
3210 REM REFERENCE: SOME COMMON BASIC PROGRAMS 3RD ED., P31
3220 REM P1=PRINCIPAL, P2=MONTHLY PAYMENTS, I1=INTEREST
3230 IF P1<25 THEN 3250
3240 P1=INT((I1*P1/12)/(1-(I1/12+1)^(-12*(P2/12)))*100+.5)/100
3250 RETURN
3260 REM SUBROUTINE TO DETERMINE TERM OF LOAN
3270 REM I1=INTEREST RATE, P1=PRINCIPAL, A1=PAYMENT AMOUNT
3280 REM REFERENCE: SOME COMMON BASIC PROGRAMS 3RD ED., P38
3290 A1=-((LOG(1-(P1*I1)/(12*A1)))/(LOG(1+(I1/12)*12)))
3300 A1=INT(A1*12+.5)
3310 RETURN
3320 END

```

---

## Critical Path Method (CPM)

---

This program calculates the time needed to complete a set of interrelated activities.

Before using the program, set up a CPM diagram and a precedence table. As you establish the network, make sure you include 'dummy' activities in the diagram. These activities have no duration, but they may be necessary to indicate precedence of some activities over others in the network.

One feature of this program allows you to revise the network by changing activity durations and costs. In this way, you can observe changes in the critical path. Depending on the degree to which you revise the network, the path may shift by adding or eliminating activities.

### Program Notes

This program currently allows 100 activities. If you want to change this, modify line 60 of the program as follows:

```
60 DIM A(I, 2), S(I), F(I), E(I,2)
```

Replace the expression I with your maximum, e.g., 15, 20, etc.

Negative slack time can exist for an activity; however, the program does not factor this into start times, end times or the critical path length.

### Example

Washoe Valves is having its state-wide sale-a-thon, a contest in which the company's three salespersons travel up Indiana, covering accounts in their territories and making as many sales as possible. At the end of their sale-a-thon, all three salespeople meet in Chicago for a recap meeting.

Nance Graham, the sales manager, wants to know when each salesperson should start the trip, how much time each will spend driving and selling and when to expect each salesperson to arrive in Chicago. Her precedence chart contains daily reimbursements to help calculate travel advances.

Activity	Nodal Sequence	Time (hours)	Cost
1. Gary drives to Terre Haute	1-2	2	30
2. Nance drives to Indianapolis	1-3	3	40
3. Lana drives to Muncie	1-4	3.5	49
4. Sell in Terre Haute	2-5	36	125
5. Sell in Indianapolis	3-6	48	320
6. Sell in Muncie	4-7	48	125
7. Gary drives to Lafayette	5-8	3	40
8. Nance drives to Chicago	6-11	5	35
9. Lana drives to Ft. Wayne, drops off valves	7-10	2	30
10. Sell in Lafayette	8-9	16	90
11. Lana drives to Chicago	9-11	4	52
12. Gary drives to Chicago	10-11	2	30

How does Nance run this program?

Answer: The minimum time needed to complete the sale-a-thon is 61 hours (the critical path length), and it will cost \$966 in travel advances.



# CRITICAL PATH METHOD

HOW MANY ACTIVITIES IN THIS NETWORK? 12

ENTER START, END NODES FOR ACTIVITY 1 ? 1 , 2  
ENTER DURATION AND COST 2 , 30

ENTER START, END NODES FOR ACTIVITY 2 ? 1 , 3  
ENTER DURATION AND COST 3 , 40

ENTER START, END NODES FOR ACTIVITY 3 ? 1 , 4  
ENTER DURATION AND COST 3.5 , 49

ENTER START, END NODES FOR ACTIVITY 4 ? 2 , 5  
ENTER DURATION AND COST 36 , 125

ENTER START, END NODES FOR ACTIVITY 5 ? 3 , 6  
ENTER DURATION AND COST 48 , 320

ENTER START, END NODES FOR ACTIVITY 6 ? 4 , 7  
ENTER DURATION AND COST 48 , 125

ENTER START, END NODES FOR ACTIVITY 7 ? 5 , 8  
ENTER DURATION AND COST 3 , 40

ENTER START, END NODES FOR ACTIVITY 8 ? 6 , 11  
ENTER DURATION AND COST 5 , 35

ENTER START, END NODES FOR ACTIVITY 9 ? 7 , 10  
ENTER DURATION AND COST 2 , 30

ENTER START, END NODES FOR ACTIVITY 10 ? 8 , 9  
ENTER DURATION AND COST 16 , 90

ENTER START, END NODES FOR ACTIVITY 11 ? 9 , 11  
ENTER DURATION AND COST 4 , 52

ENTER START, END NODES FOR ACTIVITY 12 ? 10 , 11  
ENTER DURATION AND COST 2 , 30

START NODE	END NODE	EARLY START	LATE FINISH	DURATION	SLACK	COST
1	2	0	2	2	CRITICAL	30
1	3	0	8	3	5	40
1	4	0	9	3.5	5.5	49
2	5	2	38	36	CRITICAL	125
3	6	3	56	48	5	320
4	7	3.5	57	48	5.5	125
5	8	38	41	3	CRITICAL	40
6	11	51	61	5	5	35
7	10	51.5	59	2	5.5	30
8	9	41	57	16	CRITICAL	90
9	11	57	61	4	CRITICAL	52
10	11	53.5	61	2	5.5	30

THE CRITICAL PATH LENGTH IS 61  
TOTAL COST OF THIS NETWORK= 966

DO YOU WANT TO CHANGE ANY  
ACTIVITY DURATIONS (Y/N)? N

### Practice Problems

1. Suppose Gary only spends 30 hours in Terre Haute. Will the critical path be different? Who will be able to wait before leaving, and for how long?

Answer: The critical path reduces to 56 hours. Gary can now wait 7 hours before leaving on his trip, and Lana can wait half an hour.

2. Nance may take her plane, rather than drive. The flying time to Indianapolis is half an hour, and to Chicago, the time is 45 minutes. She will have to pay a landing fee of \$5 at Indianapolis, and \$20 at Chicago, in addition to the costs shown above.

With this information, how long can she wait before leaving? What will the total cost be?

Answer: In the original network, Nance could wait 5 hours. She can now wait 11.75 hours before leaving. The total network cost is \$971.

### Program Listing

```
10 REM CRITICAL PATH METHOD (CPM) 10/21/80 10:00 AM
20 REM 'A' ARRAY = START AND END NODES FOR EACH ACTIVITY
30 REM 'S' ARRAY = EARLY START TIMES FOR EACH ACTIVITY
40 REM 'F' ARRAY = LATE FINISH TIMES FOR EACH ACTIVITY
50 REM 'E' ARRAY = DURATIONS AND COSTS OF NORMAL ACTIVITIES
60 REM 'C' ARRAY = DURATIONS AND COSTS OF CRASH ACTIVITIES
70 DIM A(100,2),S(100),F(100),E(100,2),C(100,2)
80 DEF FNR(Z1)=INT((Z1*1000+.5))/1000
90 PRINT "CRITICAL PATH METHOD"
100 PRINT
110 PRINT "HOW MANY ACTIVITIES IN THIS NETWORK";
120 INPUT N
130 FOR I=1 TO N
140 PRINT
150 PRINT "ENTER START, END NODES FOR ACTIVITY";I;
160 INPUT A(I,1),A(I,2)
170 IF A(I,2) <= A(I,1) THEN 200
190 IF A(I,2)<N THEN 260
200 PRINT " START NODE MUST BE NUMBERED LOWER"
210 PRINT " THAN END NODE, AND END NODE MUST"
220 PRINT "BE LESS THAN THE NUMBER OF ACTIVITIES."
230 PRINT " *** TRY ENTRY AGAIN ***"
240 PRINT
250 GOTO 140
260 PRINT " ENTER DURATION AND COST";
270 INPUT E(I,1),E(I,2)
280 S(I)=0
290 F(I)=0
300 NEXT I
310 REM LOOP TO FIND EARLY START TIMES FOR NETWORK
320 FOR I=1 TO N
330 IF S(A(I,2))>=S(A(I,1))+E(I,1) THEN 350
340 S(A(I,2))=S(A(I,1))+E(I,1)
350 NEXT I
```

```

360 F(A(N,2))=S(A(N,2))
370 REM LOOP TO CALCULATE LATE FINISH TIMES FOR NETWORK
380 FOR I=N TO 1 STEP -1
390 IF F(A(I,1))=0 THEN 420
400 IF F(A(I,1))>F(A(I,2))-E(I,1) THEN 420
410 GOTO 430
420 F(A(I,1))=F(A(I,2))-E(I,1)
430 NEXT I
440 C1=0
450 L=0
460 PRINT
470 REM CALCULATE SLACK TIME IN S1
480 PRINT "START    END    EARLY    LATE"
490 PRINT "NODE      NODE    START  FINISH  DURATION    SLACK    COST"
500 FOR I=1 TO N
510 PRINT A(I,1);TAB(9);A(I,2);TAB(17);S(A(I,1));TAB(24);
520 PRINT F(A(I,2));TAB(32);E(I,1);TAB(42);
530 S1=F(A(I,2))-S(A(I,1))-E(I,1)
540 IF S1>0 THEN 580
545 IF L>=F(A(I,2)) THEN 590
550 PRINT "CRITICAL";
560 L=L+E(I,1)
570 GOTO 600
590 GOTO 600
600 PRINT TAB(51);E(I,2)
610 C1=C1+E(I,2)
620 NEXT I
630 PRINT
640 PRINT "THE CRITICAL PATH LENGTH IS ";L
650 PRINT "TOTAL COST OF THIS NETWORK= ";C1
660 PRINT
670 PRINT "DO YOU WANT TO CHANGE ANY"
680 PRINT "ACTIVITY DURATIONS (Y/N)";
690 INPUT A$
700 IF A$="N" THEN 870
710 IF A$<>"Y" THEN 660
720 PRINT
730 PRINT "WHICH ACTIVITY";
740 INPUT I
750 IF I<1 THEN 720
760 IF I>N THEN 720
770 PRINT "CURRENT DURATION IS";E(I,1);", COST=";E(I,2)
780 PRINT "ENTER NEW DURATION AND COST";
790 INPUT E(I,1),E(I,2)
800 PRINT "---RECALCULATING NETWORK---"
810 PRINT
820 FOR I=1 TO N
830 S(I)=0
840 F(I)=0
850 NEXT I
860 GOTO 310
870 END

```

## Reference

Brown, Kenneth S., and ReVelle, Jack B. *Quantitative Methods for Managerial Decisions*. Reading, Mass.: Addison-Wesley, 1979.

---

## Program Evaluation and Review Technique (PERT)

---

This program calculates the minimum time needed to complete a complex project under uncertain conditions, and calculates the probability of the project's completion by a target time which you enter and can modify.

The program also calculates late start, early finish and late finish times for each activity, as well as the slack time and standard deviation of expected activity times.

Before using the program, you must first organize the project, using PERT's graphing technique or a precedence table. To use the program, you must enter the number of activities in this project, including dummy activities. For each activity, you need to enter its start and end nodes, followed by the optimistic, most likely and pessimistic duration estimates.

When you enter each activity, you must be sure each start node you enter is greater than the previous end node; if not, the program will ask you to re-enter the start and end nodes.

### Program Notes

This program is set for a maximum of 100 activities. If you want to change this, modify line 60 of the program as follows:

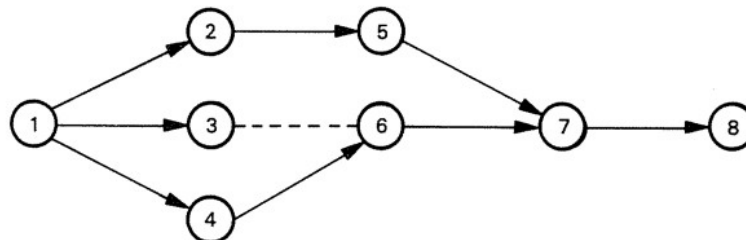
```
60 DIM A(I,2), S(I), F(I), E(I,2)
```

Replace the expression I with your maximum.

Negative slack time can exist for an activity; however, the program does not factor this into start times, end times or the critical path length.

### Example

Harriet just bought a Victorian house, advertised as a fixer-upper. She asked her contractor to provide her with three time estimates for each task involved in the remodeling. Her PERT chart and precedence table look like this:



Activity	(time given in days)				
	Start Node	End Node	Optimistic Time	Most Likely Time	Pessimistic Time
1. Scrape exterior	1	2	1	2	4
2. Remove wallpaper	1	3	2	3	5
3. Replace cabinetry	1	4	3	4	7
4. Paint exterior	2	5	2	3	6
5. (dummy activity)	3	6	0	0	0
6. Lay kitchen floor	4	6	1	2	2.5
7. Paint exterior trim	5	7	1.5	2	4
8. Paint interior walls	6	7	2	3	3
9. Refinish wood floors	7	8	2	4	5

How will she run the program? What is the minimum time needed to complete the project? What is the probability of completing it one day sooner than expected?

Answer: The minimum time to complete the project is 12.916 days. The probability of completing the remodeling in 11.916 days is approximately 12.96 percent.

#### PROGRAM EVALUATION AND REVIEW TECHNIQUE

ENTER THE NUMBER OF  
ACTIVITIES IN THIS NETWORK? 9

-----ACTIVITY 1 -----  
ENTER START NODE, END NODE? 1 , 2  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 1 , 2 , 4

-----ACTIVITY 2 -----  
ENTER START NODE, END NODE? 1 , 3  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 2 , 3 , 5

-----ACTIVITY 3 -----  
ENTER START NODE, END NODE? 1 , 4  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 3 , 4 , 7

-----ACTIVITY 4 -----  
ENTER START NODE, END NODE? 2 , 5  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 2 , 3 , 6

-----ACTIVITY 5 -----  
ENTER START NODE, END NODE? 3 , 6  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 0 , 0 , 0

-----ACTIVITY 6 -----  
ENTER START NODE, END NODE? 4 , 6  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 1 , 2 , 2.5

-----ACTIVITY 7 -----  
ENTER START NODE, END NODE? 5 , 7  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 1.5 , 2 , 4

-----ACTIVITY 8 -----  
ENTER START NODE, END NODE? 6 , 7  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 2 , 3 , 3

-----ACTIVITY 9 -----  
ENTER START NODE, END NODE? 7 , 8  
ENTER THREE TIME ESTIMATES  
FOR THIS ACTIVITY? 2 , 4 , 5

-----  
ACTIVITY 1 (NODE 1 TO NODE 2 )  
IS A NON-CRITICAL EVENT.  
EXPECTED DURATION: 2.167   STD. DEVIATION: .5  
EARLY START: 0           LATE START: 1.333  
EARLY FINISH: 2.167      LATE FINISH: 3.5  
SLACK TIME: 1.333  
-----

ACTIVITY 2 (NODE 1 TO NODE 3 )  
IS A NON-CRITICAL EVENT.  
EXPECTED DURATION: 3.167   STD. DEVIATION: .5  
EARLY START: 0           LATE START: 3.083  
EARLY FINISH: 3.167      LATE FINISH: 6.25  
SLACK TIME: 3.083  
-----

ACTIVITY 3 (NODE 1 TO NODE 4 )  
IS A CRITICAL EVENT.  
EXPECTED DURATION: 4.333   STD. DEVIATION: .667  
START NO LATER THAN: 0  
MUST BE COMPLETED BY: 4.333  
-----

ACTIVITY 4 (NODE 2 TO NODE 5 )  
IS A NON-CRITICAL EVENT.  
EXPECTED DURATION: 3.333   STD. DEVIATION: .667  
EARLY START: 2.167      LATE START: 3.5  
EARLY FINISH: 5.5       LATE FINISH: 6.833  
SLACK TIME: 1.333  
-----

ACTIVITY 5 (NODE 3 TO NODE 6 )  
IS A NON-CRITICAL EVENT.  
EXPECTED DURATION: 0   STD. DEVIATION: 0  
EARLY START: 3.167    LATE START: 6.25  
EARLY FINISH: 3.167   LATE FINISH: 6.25  
SLACK TIME: 3.083  
-----

ACTIVITY 6 (NODE 4 TO NODE 6 )  
IS A CRITICAL EVENT.  
EXPECTED DURATION: 1.917   STD. DEVIATION: .25  
START NO LATER THAN: 4.333  
MUST BE COMPLETED BY: 6.25  
-----

ACTIVITY 7 (NODE 5 TO NODE 7 )  
IS A NON-CRITICAL EVENT.  
EXPECTED DURATION: 2.25   STD. DEVIATION: .417  
EARLY START: 5.5       LATE START: 6.833  
EARLY FINISH: 7.75     LATE FINISH: 9.083  
SLACK TIME: 1.333

-----

ACTIVITY 8 (NODE 6 TO NODE 7 )  
 IS A CRITICAL EVENT.  
 EXPECTED DURATION: 2.833 STD. DEVIATION: .167  
 START NO LATER THAN: 6.25  
 MUST BE COMPLETED BY: 9.083

-----

ACTIVITY 9 (NODE 7 TO NODE 8 )  
 IS A CRITICAL EVENT.  
 EXPECTED DURATION: 3.833 STD. DEVIATION: .5  
 START NO LATER THAN: 9.083  
 MUST BE COMPLETED BY: 12.916

THE CRITICAL PATH LENGTH IS 12.916  
 PLUS OR MINUS .886159127923  
 ENTER DESIRED COMPLETION TIME(0 TO END)? 11.916  
 PROBABILITY OF COMPLETION WITH  
 DURATION OF 11.916 IS .129551983062

### Practice Problems

1. A project is charted on the precedence table below:

Activity	Optimistic Time	Most Likely Time	Pessimistic Time
1-2	.5	1	2
2-3	1	2	3
2-4	1	3	5
3-5	3	4	5
4-5	2	3	4
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	5	6	8
8-10	1	2	3
9-10	3	5	7

What is the critical path length? What is the probability of completing it within 30 weeks?

Answer: Critical path length is 27.25 weeks. The probability of completing the project within 30 weeks is .98095.

2. Here is another precedence table:

Activity	(time given in days)		
	Optimistic Time	Most Likely Time	Pessimistic Time
1-2	1	4	7
1-3	1	6	11
2-4	3	5	13
3-4	2	7	12
3-5	2	5	8
4-5	6	8	16
4-6	2	5	14
5-7	3	4	5
6-7	1	2	3

What are the slack times for the non-critical activities in this network? How many days will the project take if we want to be at least 90 percent sure of completing it on time?



Answer: Slack times: activity 1, 3 days; activity 3, 3 days; activity 5, 9.667 days; activity 7, 5 days; activity 9, 5 days. The project will take 29.725 days at the 90 percent confidence level.

### Program Listing

```

10 REM PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT) 10/3/80 10:00 AM
20 REM 'A' ARRAY = START AND END NODES FOR EACH ACTIVITY
30 REM 'S' ARRAY = EARLY START TIMES FOR EACH ACTIVITY
40 REM 'F' ARRAY = LATE FINISH TIMES FOR EACH ACTIVITY
50 REM 'E' ARRAY = EXPECTED DURATIONS AND VARIANCES OF ACTIVITIES
60 DIM A(100,2),S(100),F(100),E(100,2)
65 DEF FNR(Z1)=INT((Z1*1000+.5))/1000
70 PRINT " PROGRAM EVALUATION"
80 PRINT " AND REVIEW TECHNIQUE"
90 PRINT
100 PRINT " ENTER THE NUMBER OF"
110 PRINT "ACTIVITIES IN THIS NETWORK";
120 INPUT N
130 PRINT
140 FOR I=1 TO N
150 PRINT
160 PRINT "-----ACTIVITY";I;"-----"
170 PRINT "ENTER START NODE, END NODE";
180 INPUT A(I,1),A(I,2)
190 IF A(I,2)>=A(I,1) THEN 220
200 IF A(I,2)<N THEN 280
220 PRINT " START NODE MUST BE NUMBERED LOWER"
230 PRINT " THAN END NODE, AND END NODE MUST"
240 PRINT "BE LESS THAN THE NUMBER OF ACTIVITIES."
250 PRINT " *** TRY ENTRY AGAIN ***"
260 PRINT
270 GOTO 150
280 PRINT "ENTER THREE TIME ESTIMATES"
290 PRINT "FOR THIS ACTIVITY (A,M,B)";
300 INPUT A1,M,B
310 REM E(I,1) IS THE EXPECTED DURATION
320 E(I,1)=FNR((A1+M*4+B)/6)
330 REM E(I,2) IS THE ACTIVITY VARIANCE
340 E(I,2)=FNR((B-A1)/6)^2
350 S(I)=0
360 F(I)=0
370 NEXT I
380 REM LOOP TO FIND EARLY START TIMES FOR NETWORK
390 FOR I=1 TO N
400 IF S(A(I,2))>=S(A(I,1))+E(I,1) THEN 420
410 S(A(I,2))=S(A(I,1))+E(I,1)
420 NEXT I
430 F(A(N,2))=S(A(N,2))
440 REM LOOP TO CALCULATE LATE FINISH TIMES FOR NETWORK
450 FOR I=N TO 1 STEP -1
460 IF F(A(I,1))=0 THEN 490
470 IF F(A(I,1))>F(A(I,2))-E(I,1) THEN 490
480 GOTO 500
490 F(A(I,1))=F(A(I,2))-E(I,1)

```

```

500 NEXT I
510 V=0
520 C=0
530 L=0
540 FOR I=1 TO N
550 REM CALCULATE SLACK TIME IN S1
560 S1=F(A(I,2))-S(A(I,1))-E(I,1)
570 PRINT "-----"
580 PRINT
590 PRINT "ACTIVITY";I;"(NODE";A(I,1);"TO NODE";A(I,2);")"
600 PRINT "IS A ";
610 IF S1<=0 THEN 630
620 PRINT "NON-";
630 PRINT "CRITICAL EVENT."
640 PRINT "EXPECTED DURATION:";E(I,1);" STD. DEVIATION:";
650 PRINT SQR(E(I,2))
660 IF S1>0 THEN 740
670 PRINT " START NO LATER THAN:";S(A(I,1))
680 PRINT "MUST BE COMPLETED BY:";F(A(I,2))
690 REM ACCUMULATE PATH LENGTH IN L, VARIANCE IN V
700 IF L>=F(A(I,2)) THEN 720
710 L=F(A(I,2))
720 V=V+E(I,2)
730 GOTO 790
740 PRINT "EARLY START:";S(A(I,1));TAB(25);"LATE START:";
750 PRINT F(A(I,2))-E(I,1)
760 PRINT "EARLY FINISH:";S(A(I,1))+E(I,1);TAB(24);
770 PRINT "LATE FINISH:";F(A(I,2))
780 PRINT "SLACK TIME:";S1
790 NEXT I
800 PRINT
810 PRINT "THE CRITICAL PATH LENGTH IS ";L
820 P=SQR(V)
830 PRINT "PLUS OR MINUS";P
840 PRINT "ENTER DESIRED COMPLETION TIME(0 TO END)";
850 INPUT D
860 IF D<=0 THEN 1010
870 REM CALCULATE Z-SCORE FOR DESIRED DURATION
880 Y=(D-L)/P
890 REM CALCULATE CUMULATIVE AREA UNDER NORMAL DISTRIBUTION
900 REM REFERENCE: SOME COMMON BASIC PROGRAMS, 3RD ED. P.128
910 R=EXP(-(Y^2)/2)/2.5066282746
920 Z=Y
930 Y=1/(1+.33267*ABS(Y))
940 T=1-R*(.4361836*Y-.1201676*Y^2+.937298*Y^3)
950 IF Z>=0 THEN 970
960 T=1-T
970 PRINT "PROBABILITY OF COMPLETION WITH"
980 PRINT "DURATION OF";D;"IS";T
990 PRINT
1000 GOTO 840
1010 END

```

## References

- Brown, Kenneth S., and ReVelle, Jack B. *Quantitative Methods for Managerial Decisions*. Reading, Mass.: Addison-Wesley, 1979.
- MacCrimmon, K.R., and Ryavec, C.A. *An Analytical Study of the PERT Assumptions*. Santa Monica, Calif.: RAND Corporation, Memo RM-3408-PR, 1962.
- Moore, Franklin G., and Hendrick, Thomas E. *Production/Operations Management* (3rd ed.). Homewood, Ill.: Richard D. Irwin, 1977.

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## Transportation Algorithm

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This program allows you to allocate a resource from multiple sources of supply to multiple destinations in the most cost-efficient way. The resource can be anything: manufactured goods, personnel, etc. Linear programming can be used to solve this type of problem, but here you do not need to convert costs into an objective function; nor do you need to express data as coefficients in a series of linear equations.

To use this program, you will need to know how many sources of supply are available, as well as the supply capacity for each source. The number of demand destinations, as well as their exact demand for the resource, are also needed. Finally, you need to know the cost of transporting the resource from each source to each destination. The program will ask you for all of this information when you run it, so be sure to have it organized before entering it into the computer.

If available supply does not equal prevailing demand, the program automatically assigns the difference to a dummy source (supply less than demand) or dummy destination (supply greater than demand). Each assignment of the resource, its transportation cost per unit and its total assignment cost print out at the end of the program. If dummy variables exist in a given problem, these assignments are printed out for your information.

### Program Notes

This program allows for 10 sources and 10 destinations. If you want to change this to another maximum, modify lines 20 and 30 as follows:

```
20 DIM S(I,2), D(J,2), S1(I + J,2), C(I,J), A(I,J), Y(X,2), M(3)
30 DIM R1(I), K1(J)
```

Replace the expression I with the maximum number of sources, and replace J with the maximum number of destinations. Replace X with the maximum number of sources plus the maximum number of destinations minus 1.

You may want to change the program to receive data through DATA statements, rather than INPUT statements. If so, modify the program as shown in the "Option" section.

### Example

Smiling Jack owns an organic crop dusting operation. He has three planes which have capacities for 65, 150, and 80 gallons of insecticide each. Tomorrow, four farms need dusting. Jack calculates that, based on the sizes of the fields, they will need 100, 45, 90 and 60 gallons for the fields, respectively. Since each plane has a different capacity, and since the fields are in four different counties, Jack estimates the costs as follows for each gallon of insecticide: For plane 1 to field 1, .05; to field 2, .12; to field 3, .08; to field 4, .11. For plane 2 to field 1, .04; to field 2, .03; to field 3, .06; to field 4, .04. For plane 3 to field 1, .09; to field 2, .14; to field 3, .13; to field 4, .18. How does Jack enter this information, what are the assignments for tomorrow, and what is the total transportation cost?

Answer: The optimal assignments are: Plane 1 to field 1, where it will spray 20 gallons, and on to field 3 where it will spray 45 gallons. Plane 2 goes to field 2 first, spraying 45 gallons, then proceeds to field 3, where it uses 45 gallons of insecticide. Finally, Plane 2 goes on to field 4, where it uses the last 60 gallons of spray. Plane 3 goes to field 1 to complete the job which Plane 1 did partially. The total cost, based on those entered, is estimated at \$18.25.

NUMBER OF SOURCES? 3  
 NUMBER OF DESTINATIONS? 4  
 CAPACITY FOR SOURCE 1 65  
 CAPACITY FOR SOURCE 2 150  
 CAPACITY FOR SOURCE 3 80  
 DEMAND FROM DESTINATION 1 100  
 DEMAND FROM DESTINATION 2 45  
 DEMAND FROM DESTINATION 3 90  
 DEMAND FROM DESTINATION 4 60  
 TRANSPORTATION COSTS:  
 FROM SOURCE 1 TO DESTINATION 1 .05  
 FROM SOURCE 1 TO DESTINATION 2 .12  
 FROM SOURCE 1 TO DESTINATION 3 .08  
 FROM SOURCE 1 TO DESTINATION 4 .11  
 FROM SOURCE 2 TO DESTINATION 1 .04  
 FROM SOURCE 2 TO DESTINATION 2 .03  
 FROM SOURCE 2 TO DESTINATION 3 .06  
 FROM SOURCE 2 TO DESTINATION 4 .04  
 FROM SOURCE 3 TO DESTINATION 1 .09  
 FROM SOURCE 3 TO DESTINATION 2 .14  
 FROM SOURCE 3 TO DESTINATION 3 .13  
 FROM SOURCE 3 TO DESTINATION 4 .18

SOURCE	DEST	# UNITS	COST	TOT COST
1	1	20	.05	1
SOURCE	DEST	# UNITS	COST	TOT COST
1	3	45	.08	3.6
SOURCE	DEST	# UNITS	COST	TOT COST
2	2	45	.03	1.35
SOURCE	DEST	# UNITS	COST	TOT COST
2	3	45	.06	2.7
SOURCE	DEST	# UNITS	COST	TOT COST
2	4	60	.04	2.4
SOURCE	DEST	# UNITS	COST	TOT COST
3	1	80	.09	7.2

TOTAL COST OF SOLUTION: 18.25

DO YOU WANT TO RE-RUN THIS  
 PROGRAM WITH NEW DATA (Y/N)? N

## Practice Problems

1. The Skinheads Motorcycle Enthusiasts Society has three chapters in the state, and three imminent social engagements with competing clubs. Based on intelligence reports, the Skinheads know that they will encounter 75, 19 and 22 people respectively. Their three chapters have 35, 20 and 61 members. The mileage from Chapter 1 to location 1 is 35 miles; to location 2, 80 miles, and to location 3, 60 miles. From Chapter 2 to location 1, the distance is 90 miles; to location 2, 40 miles, and to location 3, 55 miles. From Chapter 3 to location 1, the distance is 50 miles; to destination 2, 28 miles; and to location 3, 65 miles.

How should people be assigned? How far, in miles, will everyone in the club have traveled to reach the destinations?

Answer: 35 persons from chapter 1 to location 1; 20 people from chapter 2 to destination 3; 40 people from chapter 3 to location 1; 19 people from chapter 3 to location 2, and 2 from chapter 3 to destination 3. The total miles travelled (assuming one person per bike): 4,987.

2. Given the following table, what is the optimal transportation mix? How much does it cost?

Project	Weekly Demand	Plant	Weekly Capacity
A	170	J	130
B	250	K	200
C	100	L	190

Costs:

From	To A	To B	To C
J	\$2	\$ 5	\$5
K	9	13	9
L	2	4	6

Answer: 70 units from Plant J to Project Site A; 60 units from Plant J to Project B; 100 units from Plant K to Project A; 100 units from Plant K to Project C; and 190 units from Plant L to Project B.

## Program Listing

```

10 REM TRANSPORTATION ALGORITHM REV 1.0 3/20/80 11:00 AM
20 DIM S(10,2),D(10,2),S1(20,2),C(10,10),A(10,10),Y(19,2),M(3)
30 DIM R1(10),K1(10)
40 PRINT "TRANSPORTATION ALGORITHM"
50 PRINT
60 PRINT "      NUMBER OF SOURCES";
70 INPUT S2
80 IF S2<1 THEN 60
90 PRINT "NUMBER OF DESTINATIONS";
100 INPUT D1
110 IF D1<1 THEN 90
120 REM ENTER SUPPLY CAPACITY FOR EACH SOURCE
130 T1=0
140 FOR R=1 TO S2
150 PRINT "CAPACITY FOR SOURCE";R;
160 INPUT S(R,1)
170 S(R,2)=S(R,1)
180 T1=T1+S(R,1)
190 NEXT R
200 T2=0
210 REM ENTER DEMAND FROM EACH DESTINATION
220 FOR R=1 TO D1
230 PRINT "DEMAND FROM DESTINATION";R;
240 INPUT D(R,1)
250 D(R,2)=D(R,1)

```

```

260 T2=T2+D(R,1)
270 NEXT R
280 REM LOOP TO ENTER TRANSPORTATION COSTS
290 PRINT "ENTER TRANSPORTATION COSTS:"
300 FOR R=1 TO S2
310 REM INITIALIZE ELEMENTS OF S1() ARRAY
320 S1(R,1)=0
330 S1(R,2)=0
340 FOR K=1 TO D1
350 PRINT "FROM SOURCE";R;"TO DESTINATION";K;
360 INPUT C(R,K)
370 IF C(R,K)<0 THEN 350
380 NEXT K
390 NEXT R
400 REM THE MATRIX HAS BEEN ENTERED--START FIRST SOLUTION PHASE
410 S0=0
420 D0=0
430 IF T1>=T2 THEN 480
440 REM SUPPLY MUST EQUAL DEMAND; SET UP DUMMY ROWS & COLUMNS
450 S(S2+1,1)=T2-T1
460 S(S2+1,2)=T2-T1
470 S0=1
480 IF T2=T1 THEN 510
490 D(D1+1,1)=T1-T2
500 D(D1+1,2)=T1-T2
510 D2=0
520 T3=0
530 REM START SOLUTION WITH NORTHWEST CORNER RULE
540 FOR R=1 TO S2+S0
550 REM IF SUPPLY AT ROW R EXHAUSTED, MOVE TO NEXT SOURCE
560 IF S(R,2)=0 THEN 770
570 REM ALLOCATE SUPPLY TO DEMAND
580 FOR K=1 TO D1+D0
590 REM IF DESTINATION K FILLED, INCREMENT COLUMN INDEX
600 IF D(K,2)=0 THEN 760
610 IF S(R,2)=0 THEN 760
620 IF S(R,2)<D(K,2) THEN 690
630 REM SET UP STONE SQUARE IF DEMAND <= SUPPLY
640 A(R,K)=D(K,2)
650 S(R,2)=S(R,2)-D(K,2)
660 D(K,2)=0
670 GOTO 720
680 REM SET UP STONE SQUARE IF DEMAND > SUPPLY
690 A(R,K)=S(R,2)
700 D(K,2)=D(K,2)-S(R,2)
710 S(R,2)=0
720 D2=D2+1
730 T3=T3+(A(R,K)*C(R,K))
740 S1(D2,1)=R
750 S1(D2,2)=K
760 NEXT K
770 NEXT R
780 REM CHECK SOLUTION FOR FIRST-STAGE DEGENERACY
790 IF D2=S2+S0+D1+D0-1 THEN 1140
800 REM SOLVE DEGENERATE SOLUTION

```

```

810 R=0
820 K=0
830 I=0
840 I=I+1
850 IF A(S1(I,1),S1(I,2))=D(S1(I,2),1) THEN 870
860 IF A(S1(I,1),S1(I,2))<>S(S1(I,1),1) THEN 900
870 R=S1(I,1)
880 K=S1(I,2)
890 GOTO 910
900 IF I<D2+D0 THEN 840
910 REM IF R & K ARE ZERO, THE MATRIX IS NOT DEGENERATE
920 IF R+K=0 THEN 1140
930 IF S1(I-1,2)=K THEN 960
940 K=S1(I-1,2)
950 GOTO 1000
960 IF K=D2+D0 THEN 990
970 K=K+1
980 GOTO 1000
990 K=K-1
1000 REM INSERT A NEW STONE SQUARE IN THE SOLUTION
1010 IF K>S1(I,2) THEN 1030
1020 I=I-1
1030 FOR J=D1+D0 TO I+1 STEP-1
1040 S1(J,1)=S1(J-1,1)
1050 S1(J,2)=S1(J-1,2)
1060 NEXT J
1070 S1(I,1)=R
1080 S1(I,2)=K
1090 Y(I,1)=0
1100 Y(I,2)=0
1110 D2=D2+1
1120 GOTO 800
1130 REM CALCULATE RIM VALUES
1140 FOR I=1 TO D1+D0
1150 K1(I)=-9E9
1160 NEXT I
1170 FOR I=1 TO S2+S0
1180 R1(I)=-9E9
1190 NEXT I
1200 R1(1)=0
1210 K1(S1(1,2))=C(S1(1,1),S1(1,2))
1220 R=1
1230 K=1
1240 I=1
1250 I=I+1
1260 IF K1(S1(I,2))<>-9E9 THEN 1300
1270 IF R1(S1(I,1))=-9E9 THEN 1330
1280 K1(S1(I,2))=C(S1(I,1),S1(I,2))-R1(S1(I,1))
1290 K=K+1
1300 IF R1(S1(I,1))<>-9E9 THEN 1330
1310 R1(S1(I,1))=C(S1(I,1),S1(I,2))-K1(S1(I,2))
1320 R=R+1
1330 IF I<D2 THEN 1250
1340 IF K<D1+D0 THEN 1240
1350 IF R<S2+S0 THEN 1230

```



```

1360 I=1
1370 M(1)=0
1380 REM FIND AN ELEMENT WITH THE LOWEST INDEX
1390 FOR R=1 TO S2+S0
1400 FOR K=1 TO D1+D0
1410 IF R<>S1(I,1) THEN 1450
1420 IF K<>S1(I,2) THEN 1450
1430 I=I+1
1440 GOTO 1490
1450 IF M(1)<C(R,K)-R1(R)-K1(K) THEN 1490
1460 M(1)=C(R,K)-R1(R)-K1(K)
1470 M(2)=R
1480 M(3)=K
1490 NEXT K
1500 NEXT R
1510 IF M(1)>=0 THEN 2790
1520 REM FIND A CLOSED PATH FROM SQUARE AT ROW R, COL. K
1530 Y(1,1)=M(2)
1540 Y(1,2)=M(3)
1550 Q=1
1560 IF M(2)=S2+S0 THEN 1960
1570 REM M0=CURRENT ROW TO SEARCH ON; M1=START COLUMN TO SEARCH ON
1580 M0=Y(Q,1)
1590 M1=1
1600 REM START ROW SEARCH
1610 I=0
1620 I=I+1
1630 IF S1(I,1)>M0 THEN 1670
1640 IF S1(I,1)<M0 THEN 1660
1650 IF S1(I,2)>=M1 THEN 1720
1660 IF I<D2 THEN 1620
1670 IF Q<>1 THEN 1700
1680 PRINT "MATRIX IS DEGENERATE"
1690 GOTO 2410
1700 REM AT THIS POINT, NO ROW NEIGHBORS EXIST
1710 GOTO 1830
1720 REM MAKE SURE V(I) IS NOT ALREADY ON THE CLOSED PATH
1730 X0=0
1740 FOR J=1 TO Q
1750 IF S1(I,1)<>Y(J,1) THEN 1780
1760 IF S1(I,2)<>Y(J,2) THEN 1780
1770 X0=1
1780 NEXT J
1790 IF X0=0 THEN 1890
1800 M1=S1(I,1)+1
1810 IF M1<=D1+D0 THEN 1660
1820 REM ROW SEARCH FAILED; SET NEXT COORDINATES FOR COLUMN SEARCH
1830 P=Y(Q,2)
1840 P1=Y(Q,1)+1
1850 Y(Q,1)=0
1860 Y(Q,2)=0
1870 Q=Q-1
1880 GOTO 2000
1890 Q=Q+1
1900 Y(Q,1)=S1(I,1)

```

```

1910 Y(Q,2)=S1(I,2)
1920 IF Q<=2 THEN 1950
1930 REM IF PATH CLOSES ON A ROW SEARCH, EXIT SEARCH ROUTINE
1940 IF Y(Q,2)=M(3) THEN 2340
1950 M1=Y(Q,2)+1
1960 REM COLUMN SEARCH AREA
1970 REM P=COLUMN NUMBER TO SEARCH ON; P1=STARTING ROW FOR SEARCH
1980 P=Y(Q,2)
1990 P1=1
2000 K=0
2010 K=K+1
2020 IF S1(K,1)<P1 THEN 2040
2030 IF S1(K,2)=P THEN 2120
2040 IF K<D2 THEN 2010
2050 REM COLUMN SEARCH FAILURE; SET NEW COORDINATES FOR ROW SEARCH
2060 M0=Y(Q,1)
2070 M1=Y(Q,2)+1
2080 Y(Q,1)=0
2090 Y(Q,2)=0
2100 Q=Q-1
2110 GOTO 1610
2120 X0=0
2130 REM LOOKUP ROUTINE: CHECK FOR ALREADY-USED STONE SQUARE
2140 FOR J=1 TO Q
2150 IF S1(K,1)<>Y(J,1) THEN 2180
2160 IF S1(K,2)<>Y(J,2) THEN 2180
2170 X0=1
2180 NEXT J
2190 IF X0=0 THEN 2250
2200 P1=S1(K,1)+1
2210 IF P1<=S2+S0 THEN 2040
2220 GOTO 2050
2230 REM A UNIQUE STONE SQUARE WAS FOUND---
2240 REM ADD IT TO THE CLOSED PATH ARRAY.
2250 Q=Q+1
2260 Y(Q,1)=S1(K,1)
2270 Y(Q,2)=S1(K,2)
2280 REM IF PATH CLOSES ON COLUMN SEARCH, EXIT SEARCH ROUTINE
2290 IF Y(Q,1)=M(2) THEN 2340
2300 P1=Y(Q,1)+1
2310 M0=Y(Q,1)
2320 M1=Y(Q,2)+1
2330 GOTO 1610
2340 REM FIND LOWEST-ALLOCATION STONE SQUARE ON CLOSED PATH
2350 X0=A(Y(2,1),Y(2,2))
2360 FOR K=4 TO Q STEP 2
2370 IF X0<=A(Y(K,1),Y(K,2)) THEN 2390
2380 X0=A(Y(K,1),Y(K,2))
2390 NEXT K
2400 REM ALTERNATELY ADD & SUBTRACT X0 FROM CLOSED PATH ALLOCATIONS
2410 P=0
2420 FOR K=1 TO Q
2430 IF K/2=INT(K/2) THEN 2460
2440 A(Y(K,1),Y(K,2))=A(Y(K,1),Y(K,2))+X0
2450 GOTO 2630

```

```

2460 A(Y(K,1),Y(K,2))=A(Y(K,1),Y(K,2))-X0
2470 IF A(Y(K,1),Y(K,2))>0 THEN 2630
2480 REM DELETE ANY SQUARES WITH A ZERO ALLOCATION
2490 I=0
2500 P=P+1
2510 REM IF P>1, MATRIX WILL BE DEGENERATE IF SQUARE IS DELETED;
      SKIP IT
2520 IF P>1 THEN 2630
2530 I=I+1
2540 IF S1(I,1)<>Y(K,1) THEN 2530
2550 IF S1(I,2)<>Y(K,2) THEN 2530
2560 FOR J=I TO D2
2570 S1(J,1)=S1(J+1,1)
2580 S1(J,2)=S1(J+1,2)
2590 NEXT J
2600 S1(D2,1)=0
2610 S1(D2,2)=0
2620 D2=D2-1
2630 NEXT K
2640 REM INSERT NEW STONE SQUARE FROM 1ST ELEMENT OF Y()
2650 I=0
2660 I=I+1
2670 IF Y(1,1)>S1(I,1) THEN 2660
2680 IF Y(1,1)<S1(I,1) THEN 2700
2690 IF Y(1,2)>S1(I,2) THEN 2660
2700 FOR J=D2 TO I STEP -1
2710 S1(J+1,1)=S1(J,1)
2720 S1(J+1,2)=S1(J,2)
2730 NEXT J
2740 S1(I,1)=Y(1,1)
2750 S1(I,2)=Y(1,2)
2760 D2=D2+1
2770 REM END OF RE-ALLOCATION; REITERATE MODI CHECK
2780 GOTO 1140
2790 REM DISPLAY RESULTS AND COST OF SOLUTION
2800 PRINT
2810 IF D0+S0=0 THEN 2870
2820 PRINT "*** UNBALANCED SOLUTION ***"
2830 IF D0=0 THEN 2850
2840 PRINT "EXCESS SUPPLY (";D(D1+D0,1);") ASSIGNED TO DESTINATION"
      ;D1+D0
2850 IF S0=0 THEN 2870
2860 PRINT "EXCESS DEMAND (";S(S2+S0,1);") ASSIGNED TO SOURCE";S2+S0
2870 X0=0
2880 FOR I=1 TO D2
2890 PRINT "SOURCE ", "DEST ", "# UNITS ", "COST ", "TOT COST"
2900 PRINT S1(I,1),S1(I,2),A(S1(I,1),S1(I,2)),C(S1(I,1),S1(I,2)),
2910 J=C(S1(I,1),S1(I,2))*A(S1(I,1),S1(I,2))
2920 IF J>0 THEN 2950
2930 PRINT "DUMMY"
2940 GOTO 2970
2950 X0=X0+J
2960 PRINT J
2970 NEXT I
2980 PRINT

```

```

2990 PRINT "TOTAL COST OF SOLUTION:";X0
3000 PRINT
3010 PRINT
3020 PRINT "DO YOU WANT TO RE-RUN THIS"
3030 PRINT "PROGRAM WITH NEW DATA (Y/N)";
3040 INPUT X0$
3050 IF X0$="Y" THEN 50
3060 END

```

### Option

If you want to avoid using INPUT statements for data entry, you can change the program to read input from DATA statements. This is especially useful if you intend to enter a large transportation problem, or if you want to run the program repeatedly with slightly different data without re-entering the supply, demand and cost figures. Modify the statements below to allow for this feature.

```

151 REM THIS DATA SHOWN TO SOLVE PROBLEM #1. PUT YOUR SUPPLY
    DATA HERE.
152 DATA 65,150,80
160 READ S(R,1)
165 PRINT S(R,1)
170 S(R,2)=S(R,1)
180 T1=T1+S(R,1)
190 NEXT R
200 T2=0
210 REM READ DATA LIST OF DEMAND FROM EACH DESTINATION
220 FOR R=1 TO D1
230 PRINT "DEMAND FROM DESTINATION";R;
231 REM PUT DEMAND DATA HERE.
232 DATA 100,45,90,60
240 READ D(R,1)
245 PRINT D(R,1)
250 D(R,2)=D(R,1)
260 T2=T2+D(R,1)
270 NEXT R
280 REM LOOP TO READ TRANSPORTATION COSTS
290 PRINT "TRANSPORTATION COSTS:"
300 FOR R=1 TO S2
310 REM INITIALIZE ELEMENTS OF S1() ARRAY
320 S1(R,1)=0
330 S1(R,2)=0
340 FOR K=1 TO D1
350 PRINT "FROM SOURCE";R;"TO DESTINATION";K;
351 REM PUT TRANSPORTATION COST DATA HERE.
352 DATA .05,.12,.08,.11,.04,.03,.06,.04,.09,.14,.13,.18
360 READ C(R,K)
365 PRINT C(R,K)
380 NEXT K
390 NEXT R

```

Also delete lines 2980 through 3060.

## References

- Chase, Richard B., and Aquilano, Nicholas J. *Production and Operations Management*. Homewood, Ill.: Richard D. Irwin, Inc., 1977.
- Levin, Richard I., and Kirkpatrick, Charles A. *Quantitative Approaches to Management* (3rd ed.). New York: McGraw-Hill, 1975.

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## Swedish Machine (Queuing Theory)

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This is the classic problem where you have X repairmen servicing Y machines. The machines are statistically identical; their times-to-failure follow the exponential law, characterized by the mean time-to-failure. The repairmen are also statistically identical; their repair completion times follow the exponential law and are characterized by its expected value. All elements are mutually independent.

This program is especially useful in that it can provide a cash flow analysis that can help project the feasibility of a particular machines-to-repairmen ratio, given the repairmen's wages, machine revenue and overhead costs.

To use the program, enter the number of machines, the mean time-to-failure for a machine, the number of repairmen, and the mean time to repair a machine. You may use any time unit base you wish, as long as you use the same time units throughout the program. A variety of system characteristics are output. If you choose to obtain a cash flow analysis, you must also enter the cost for one repairman per unit of time, the cost of possessing a machine (overhead) per unit of time, and the amount of revenue produced by a machine per unit of time. You may use any monetary unit you wish (pennies, dollars, thousands of dollars, pesos, whatever).

### Example

Ace Laundromat has a total of 50 machines operating throughout the city. The machines have a mean time-to-failure of 300 hours, and there are currently 3 repairmen. Each repairman requires 24 hours to repair a machine. At any time, how many machines can be expected to be operating? How many machines are being repaired? How many are waiting to be repaired? What is the mean down time per machine? How many repairmen are idle? Repairmen cost \$5.25 per hour (including fringe benefits, etc.). Cost of possessing a machine is the overhead involved, such as lease or purchase payments, insurance payments, pro-rated administrative expenses, depreciation expense, etc. In this example the cost of possessing a machine is \$36.00 per month, or \$0.05 per hour. What cash flow do the machines generate if they each produce revenues of \$0.50 per hour?

Answer: 37 machines can be expected to be operational at any time, 3 are being repaired, and 10 are waiting to be repaired. The mean down time per machine is about 105 hours. The 50 machines will produce an average revenue of \$0.271997 per hour.

### SWEDISH MACHINE

INPUT THE NUMBER OF MACHINES; COUNT MUST EXCEED ONE.

? 50

INPUT MEAN TIME-TO-FAILURE FOR A MACHINE? 300

INPUT NUMBER OF REPAIRMEN? 3

INPUT MEAN REPAIR TIME (PER MACHINE) FOR A REPAIRMAN? 24

THE SYSTEM IS SAID TO BE 'IN STATE OF J' IF J MACHINES ARE IN A FAILED CONDITION. THE STATIONARY PROBABILITY DISTRIBUTION OVER THE POSSIBLE STATES, 0 THRU 50 , AND OTHER CHARACTERISTICS OF INTEREST, FOLLOW.

STATE	PROBABILITY	NO. MACHINES OPERATING	NO. MACHINES WAITING	NO. REPAIRMEN IDLE
0	.001966	50	0	3
1	.007863	49	0	2
2	.01541	48	0	1
3	.019723	47	0	0
4	.024717	46	1	0
5	.030316	45	2	0
6	.036375	44	3	0
7	.042675	43	4	0
8	.048929	42	5	0
9	.054795	41	6	0
10	.059903	40	7	0
11	.06389	39	8	0
12	.066438	38	9	0
13	.067317	37	10	0
14	.066412	36	11	0
15	.063749	35	12	0
16	.059493	34	13	0
17	.053934	33	14	0
18	.047457	32	15	0
19	.040492	31	16	0
20	.03347	30	17	0
21	.026773	29	18	0
22	.020702	28	19	0
23	.015456	27	20	0
24	.011127	26	21	0
25	.007714	25	22	0
26	.005142	24	23	0
27	.003291	23	24	0
28	.002018	22	25	0
29	.001184	21	26	0
30	.000663	20	27	0
31	.000354	19	28	0
32	.000179	18	29	0
33	.000086	17	30	0
34	.000039	16	31	0
35	.000017	15	32	0
36	.000007	14	33	0
37	.000003	13	34	0
38	.000001	12	35	0
39	0	11	36	0
40	0	10	37	0
41	0	9	38	0
42	0	8	39	0
43	0	7	40	0
44	0	6	41	0
45	0	5	42	0
46	0	4	43	0
47	0	3	44	0
48	0	2	45	0
49	0	1	46	0
50	0	0	47	0

TO CONTINUE, PRESS 'RETURN'?

## SYSTEM CHARACTERISTICS

-----

NO. OF MACHINES = 50  
MEAN TIME-TO-FAILURE PER MACHINE = 300 TIME UNITS  
NO. OF REPAIRMEN = 3  
MEAN REPAIR-TIME PER REPAIRMAN = 24 TIME UNITS  
NUMBER OF MACHINES PER REPAIRMAN = 16.666667

PROB. (SERVICE SYSTEM IS EMPTY) = .001966  
PROB. (NO MACHINES ARE WAITING FOR SERVICE) = .044962

EXP. NO. OF MACHINES OPERATING = 37.043994  
EXP. NO. OF INACTIVE MACHINES = 12.956006  
EXP. NO. OF MACH. IN WAITING LINE = 9.9968  
EXP. NO. OF MACH. IN A NON-EMPTY 'WAITING LINE' = 10.467437  
MEAN DOWN-TIME PER MACHINE = 104.92394 TIME UNITS  
MEAN WAITING TIME PER MACHINE = 80.958873 TIME UNITS  
EXP. NO. OF REPAIRMEN IDLE = .115163

TO CONTINUE, PRESS 'RETURN'?

'COEFFICIENT OF LOSS' FOR MACHINES = FRACTION OF TIME A  
MACHINE IS 'DOWN' AS A CONSEQUENCE OF THE SYSTEM  
CHARACTERISTICS = .199936

'COEFFICIENT OF LOSS' FOR REPAIRMEN = FRACTION OF TIME A  
REPAIRMAN IS IDLE AS A CONSEQUENCE OF THE SYSTEM  
CHARACTERISTICS = .038388

TYPE 1 FOR CASH FLOW ANALYSIS  
2 TO HALT

? 1  
THIS ANALYSIS ASSUMES THAT REPAIRMEN ARE PAID 'A' MONETARY  
UNITS PER UNIT TIME, THAT THE FIXED COST OF POSSESSING EACH  
MACHINE IS 'B' MONETARY UNITS PER UNIT TIME, AND THAT A MACHINE,  
WHEN OPERATING, IS CAPABLE OF PRODUCING 'C' UNITS OF REVENUE  
PER UNIT TIME.

INPUT THE REPAIRMAN-COST PER UNIT TIME, 'A'? 5.25

INPUT THE FIXED COST PER UNIT TIME, 'B', OF POSSESSING  
A MACHINE? .05

INPUT THE AMOUNT OF REVENUE A WORKING MACHINE PRODUCES,  
PER UNIT OF (OPERATING) TIME? .5

THE AVERAGE CASH FLOW GENERATED BY THE COMBINATION OF 50  
MACHINE(S) MAINTAINED BY 3 REPAIRMEN IS .271997 MONETARY  
UNITS, PER UNIT TIME.



### Practice Problem

In the above example, suppose Ace invested \$65.00 per machine to retrofit them with heavy duty motors, raising their mean times-to-failure to 305 hours. What cash flow will the machines produce? How much time must pass before Ace has recovered their \$3,250.00 investment?

Answer: If retrofit, the machines will produce an average revenue of \$0.525136 per hour. The investment will be recovered within 18 months.

### Program Listing

```
10 PRINT "SWEDISH MACHINE"
20 REM -- CHANGE DIMENSION OF Q() TO MAXIMUM NUMBER OF MACHINES + 1
30 DIM Q(100)
40 PRINT
50 PRINT "INPUT THE NUMBER OF MACHINES; COUNT MUST EXCEED ONE."
60 INPUT N
70 PRINT "INPUT MEAN TIME-TO-FAILURE FOR A MACHINE";
80 INPUT F1
90 F=1/F1
100 PRINT "INPUT NUMBER OF REPAIRMEN";
110 INPUT M
120 PRINT "INPUT MEAN REPAIR TIME (PER MACHINE) FOR A REPAIRMAN";
130 INPUT R1
140 R=1/R1
150 PRINT
160 REM -- INITIALIZE VARIABLES
170 FOR I=1 TO N+1
180 Q(I)=0
190 NEXT I
200 Q(1)=1
210 E1=0
220 E2=0
230 E3=0
240 P0=0
250 REM -- LOOP TO CALCULATE PROBABILITIES FOR EACH MACHINE
260 S=Q(1)
270 FOR J=0 TO N-1
280 REM -- K=MIN(J+1,M)
290 K=M
300 IF J+1>M THEN 320
310 K=J+1
320 Q(J+2)=(N-J)*F*Q(J+1)/K/R
330 S=S+Q(J+2)
340 NEXT J
350 IF Q(1)<>1 THEN 380
360 Q(1)=1/S
370 GOTO 260
380 PRINT
390 PRINT "THE SYSTEM IS SAID TO BE 'IN STATE J' IF J MACHINES ARE"
400 PRINT "IN A FAILED CONDITION. THE STATIONARY PROBABILITY DISTRI-"
410 PRINT "BUTION OVER THE POSSIBLE STATES, 0 THRU";N;"", AND OTHER"
420 PRINT "CHARACTERISTICS OF INTEREST, FOLLOW."
430 PRINT
440 PRINT "STATE","PROBABILITY",
442 PRINT "NO. MACHINES","NO. MACHINES","NO. REPAIRMEN"
450 PRINT "",""," OPERATING"," WAITING"," IDLE"
```

```

460 FOR J=1 TO N+1
470 O=N-J+1
480 W=J-M-1
490 IF W>0 THEN 520
500 W=0
510 P0=P0+Q(J)
520 I=M-J+1
530 IF I>0 THEN 550
540 I=0
550 IF I<M THEN 570
560 I=M
570 PRINT " ";J-1,Q(J),O,W,I
580 E1=E1+W*Q(J)
590 E2=E2+I*Q(J)
600 E3=E3+O*Q(J)
610 NEXT J
620 PRINT
630 PRINT "TO CONTINUE, PRESS 'RETURN'";
640 INPUT Z$
650 PRINT
660 PRINT TAB(15);"SYSTEM CHARACTERISTICS"
670 PRINT TAB(15);"-----"
680 PRINT "NO. OF MACHINES =" ;N
690 PRINT "MEAN TIME-TO-FAILURE PER MACHINE =" ;F1;"TIME UNITS"
700 PRINT "NO. OF REPAIRMEN =" ;M
710 PRINT "MEAN REPAIR-TIME PER REPAIRMAN =" ;R1;"TIME UNITS"
720 PRINT "NUMBER OF MACHINES PER REPAIRMAN =" ;N/M
730 PRINT
740 PRINT "PROB. (SERVICE SYSTEM IS EMPTY) =" ;Q(1)
750 PRINT "PROB. (NO MACHINES ARE WAITING FOR SERVICE) =" ;P0
760 PRINT
770 PRINT "EXP. NO. OF MACHINES OPERATING =" ;E3
780 PRINT "EXP. NO. OF INACTIVE MACHINES =" ;N-E3
790 PRINT "EXP. NO. OF MACH. IN WAITING LINE =" ;E1
800 PRINT "EXP. NO. OF MACH. IN A NON-EMPTY 'WAITING LINE' =" ;E1/(1-P0)
810 PRINT "MEAN DOWN-TIME PER MACHINE =" ;(N-E3)*F1/E3;"TIME UNITS"
820 PRINT "MEAN WAITING TIME PER MACHINE =" ;E1*F1/E3;"TIME UNITS"
830 PRINT "EXP. NO. OF REPAIRMEN IDLE =" ;E2
840 PRINT
850 PRINT "TO CONTINUE, PRESS 'RETURN'";
860 INPUT Z$
870 PRINT "'COEFFICIENT OF LOSS' FOR MACHINES = FRACTION OF TIME A"
880 PRINT " MACHINE IS 'DOWN' AS A CONSEQUENCE OF THE SYSTEM"
890 PRINT " CHARACTERISTICS =" ;E1/N
900 PRINT
910 PRINT "'COEFFICIENT OF LOSS' FOR REPAIRMEN = FRACTION OF TIME A"
920 PRINT " REPAIRMAN IS IDLE AS A CONSEQUENCE OF THE SYSTEM"
930 PRINT " CHARACTERISTICS =" ;E2/M
940 PRINT
950 PRINT "TYPE 1 FOR CASH FLOW ANALYSIS"
960 PRINT " 2 TO HALT"
970 INPUT Q1
980 IF Q1=2 THEN 1250
990 PRINT "THIS ANALYSIS ASSUMES THAT REPAIRMEN ARE PAID 'A' MONETARY"
1000 PRINT "UNITS PER UNIT TIME, THAT THE FIXED COST OF POSSESSING"

```

```

1010 PRINT "EACH MACHINE IS 'B' MONETARY UNITS PER UNIT TIME, AND THAT"
1020 PRINT "A MACHINE, WHEN OPERATING, IS CAPABLE OF PRODUCING 'C'"
1030 PRINT "UNITS OF REVENUE PER UNIT TIME."
1040 PRINT
1050 PRINT "INPUT THE REPAIRMAN-COST PER UNIT TIME, 'A'";
1060 INPUT A
1070 PRINT
1080 PRINT "INPUT THE FIXED COST PER UNIT TIME, 'B', OF POSSESSING"
1090 PRINT "A MACHINE";
1100 INPUT B
1110 PRINT
1120 PRINT "INPUT THE AMOUNT OF REVENUE A WORKING MACHINE PRODUCES,"
1130 PRINT "PER UNIT OF (OPERATING) TIME";
1140 INPUT C
1150 PRINT
1160 D=C*E3-A*M-B*N
1170 PRINT "THE AVERAGE CASH FLOW GENERATED BY THE COMBINATION OF";N
1180 PRINT "MACHINE(S) MAINTAINED BY";M;"REPAIR";
1190 IF M>1 THEN 1220
1200 PRINT "MAN";
1210 GOTO 1230
1220 PRINT "MEN";
1230 PRINT " IS";D;"MONETARY"
1240 PRINT "UNITS, PER UNIT TIME."
1250 END

```

---

## Markov Analysis

---

This program calculates the future changes, over time, in a given variable based on its current movement. Management scientists adopted this analysis, using it mostly as a simulation technique for analyzing competitors in the marketplace. Markov analysis has many other applications, however, as illustrated by the examples below.

To use the program, first enter how many states of nature are under consideration. The second entry is optional. If you want to see changes occur over time from stage to stage, you must enter the current population proportion vector. If you are only interested in long-run steady-state equilibrium, the program will seed the vector with equal probabilities. The number of elements in this vector equals the states of nature.

The program then asks you to enter each cell of the transition probabilities matrix ( $N \times N$ , where  $N$  = states of nature). For each cell, enter a transition probability, ranging  $0 \leq p \leq 1$ . The sum of the probabilities entered for each row should always add up to 1. Once you have entered the entire matrix, you have the option of looking at each future period or letting the computer calculate the transition matrix at equilibrium.

The program displays the equilibrium vector, the period at which equilibrium was reached, and the first passage times for each state of nature. First passage times will not print for recurrent or null-recurrent states.

### Program Notes

This program allows for a maximum of 12 states of nature. You can change this by modifying line 20 of this program:

```
20 DIM V1(I), T(I,I), V2(I)
```

Replace I with your maximum (e.g., 15, 20, or 25).

If you have large matrices to enter, or if you want to repeatedly run this program with mostly the same data, you can modify the program to accept data through DATA statements, as shown in the "Option" section.

### Example

Caffrey's Hardware wants to analyze its accounts receivable in order to estimate its cash flow from credit customers. The company has three aging categories: current, 45-89 days, and 90-plus days past due. Customers in this last category are eventually written off as uncollectable accounts.

The latest aging analysis shows that, for each dollar of accounts receivable outstanding, \$ .60 is current, \$ .33 is 45-89 days old, and \$ .07 is 90-plus days old. Further analysis shows that accounts in the Current category have a 38% chance of being paid in the next month, 45% of all current accounts will remain current, and 17% will be 45-89 days old. Accounts in the 45-89 days category stand a 65% chance of paying all back payments, a 25% chance of paying only the late installment, and a 5% chance of becoming 90-plus days overdue. Of the accounts in the 90-plus category, there is a 25% chance they will be paid and a 75% chance they will become bad debts.

The paid and bad debt categories are "absorbing" states, in that the probability of a paid item remaining paid is assumed to be 100%. The same is true for bad debts. These are called absorbing states because all accounts outstanding now will, eventually, be paid up or written off. How much of accounts receivable will be collected? How much will be written off?

On the printout below, the paid category and bad debt category have absorbed all outstanding debts. Caffrey can expect about 91% of his accounts to be paid, and 9% to be written off.

# MARKOV ANALYSIS

HOW MANY STATES OF NATURE? 5

IS THE POPULATION PROPORTION VECTOR KNOWN (Y/N)? Y

ENTER VECTOR ELEMENT 1 ? 0  
 ENTER VECTOR ELEMENT 2 ? .6  
 ENTER VECTOR ELEMENT 3 ? .33  
 ENTER VECTOR ELEMENT 4 ? .07  
 ENTER VECTOR ELEMENT 5 ? 0

ENTER ELEMENT IN ROW 1 COLUMN 1 ? 1  
 ENTER ELEMENT IN ROW 1 COLUMN 2 ? 0  
 ENTER ELEMENT IN ROW 1 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 1 COLUMN 4 ? 0  
 ENTER ELEMENT IN ROW 1 COLUMN 5 ? 0

ENTER ELEMENT IN ROW 2 COLUMN 1 ? .38  
 ENTER ELEMENT IN ROW 2 COLUMN 2 ? .45  
 ENTER ELEMENT IN ROW 2 COLUMN 3 ? .17  
 ENTER ELEMENT IN ROW 2 COLUMN 4 ? 0  
 ENTER ELEMENT IN ROW 2 COLUMN 5 ? 0

ENTER ELEMENT IN ROW 3 COLUMN 1 ? .65  
 ENTER ELEMENT IN ROW 3 COLUMN 2 ? .25  
 ENTER ELEMENT IN ROW 3 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 3 COLUMN 4 ? .05  
 ENTER ELEMENT IN ROW 3 COLUMN 5 ? 0

---PROBABILITIES DO NOT ADD UP TO 1.0---

TRY ENTERING THE ROW AGAIN.

ENTER ELEMENT IN ROW 3 COLUMN 1 ? .65  
 ENTER ELEMENT IN ROW 3 COLUMN 2 ? .25  
 ENTER ELEMENT IN ROW 3 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 3 COLUMN 4 ? .1  
 ENTER ELEMENT IN ROW 3 COLUMN 5 ? 0

ENTER ELEMENT IN ROW 4 COLUMN 1 ? .25  
 ENTER ELEMENT IN ROW 4 COLUMN 2 ? 0  
 ENTER ELEMENT IN ROW 4 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 4 COLUMN 4 ? 0  
 ENTER ELEMENT IN ROW 4 COLUMN 5 ? .75

ENTER ELEMENT IN ROW 5 COLUMN 1 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 2 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 4 ? 0

ENTER ELEMENT IN ROW 4 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 4 COLUMN 4 ? 0  
 ENTER ELEMENT IN ROW 4 COLUMN 5 ? .75

ENTER ELEMENT IN ROW 5 COLUMN 1 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 2 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 3 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 4 ? 0  
 ENTER ELEMENT IN ROW 5 COLUMN 5 ? 1

DO YOU WANT TO OBSERVE EACH  
PERIOD UNDER ANALYSIS (Y/N)? Y

POPULATION PROPORTION  
VECTOR AT PERIOD 2 IS:

.46  
.3525  
.102  
.033  
.0525

.  
.  
.

POPULATION PROPORTION  
VECTOR AT PERIOD 17 IS:

.9058  
0  
0  
0  
.0946

EQUILIBRIUM REACHED AT PERIOD 17  
VECTOR AT EQUILIBRIUM:

.9058  
0  
0  
0  
.0946

STOPPED IN 860

DO YOU WANT TO RE-RUN THIS  
PROGRAM WITH DIFFERENT DATA (Y/N)? N

### Practice Problems

1. A survey by Hanley, Ohio city planners shows recent commuting trends. Citizens were polled to find out if they carpool, take the bus or drive alone to and from work. Presently, 43% of commuters drive their cars alone, 30% carpool and 27% take the bus to work. The city wants to know how these patterns will change over the coming months in order to increase or decrease their bus fleet. The survey shows that 65% of those who drive alone will continue to do so. 20% of this group said they would carpool, and 15% would take the bus if gas prices continue to rise. 25% of carpoolers say that they find driving alone is preferable, and that they will switch back to it. 55% of carpoolers say that they will continue to carpool, and the remaining 20% will switch to the bus.

12% of bus riders will switch back to driving alone. 13% of bus riders say they will switch to carpooling, and 75% say they will continue to ride the bus. What will the commuting mix be 6 months from now? What will it look like at equilibrium?

Answer: In the sixth month, 35.5% will be driving alone, 26.66% will be carpooling and 39.86% will be taking the bus. At equilibrium, 32.86% will be driving alone, 26.4% will be carpooling and 40.83% will be riding the bus.

2. Rita's Rent-A-Car competes with two other rental agencies at Manteca Airport. In the past month, Rita's kept 85% of its customers from the previous month, lost 5% of its business to competitor A, and lost 10% to competitor B. Competitor A retained 90% of its customers while losing 10% to Competitor B. Competitor B retained 75% of its customers, while losing 15% to Competitor A, and 10% to Rita's. What are the equilibrium market shares, assuming no known proportion vector? How long, in months, does it take for a customer to return to Rita's to rent a car after having taken his/her business elsewhere?

Answer: Equilibrium shares: Rita's, 19.1%; Competitor A, 52.4%; Competitor B, 28.63%. On the average, it takes 5.24 months for a patron of either competitor to switch to Rita's.

### Program Listing

```

10 REM MARKOV ANALYSIS--VERS OF 3/28/80, 10:00 AM
20 DIM V1(12),T(12,12),V2(12)
30 REM V1() ARRAY--POPULATION PROPORTION VECTOR
40 REM T() ARRAY--TRANSITION PROBABILITIES MATRIX
50 REM V2() ARRAY--SCRATCH FOR VECTOR ARRAY
60 DEF FNR(Z)=INT((Z*10000+.5))/10000
70 PRINT "MARKOV ANALYSIS"
80 PRINT
90 N=1
100 PRINT "HOW MANY STATES OF NATURE";
110 INPUT S
120 PRINT
130 PRINT "IS THE POPULATION PROPORTION VECTOR KNOWN (Y/N)";
140 INPUT A$
150 IF A$="Y" THEN 220
160 IF A$<>"N" THEN 130
170 REM IF VECTOR UNKNOWN, ASSIGN EQUAL PROBABILITIES TO EACH
    STATE
180 FOR I=1 TO S
190 V1(I)=FNR(1/S)
200 NEXT I
210 GOTO 280
220 REM LOOP TO ENTER POPULATION PROPORTIONS
230 PRINT
240 FOR I=1 TO S
250 PRINT "          ENTER VECTOR ELEMENT";I;
260 INPUT V1(I)
270 NEXT I
280 REM ENTER TRANSITION MATRIX (I BY J ARRAY)
290 PRINT
300 FOR I=1 TO S
310 K=0
320 FOR J=1 TO S
330 PRINT "ENTER ELEMENT IN ROW";I;"COLUMN";J;
340 INPUT T(I,J)
350 K=K+T(I,J)
360 NEXT J
370 IF K=1 THEN 410
380 PRINT "----PROBABILITIES DO NOT ADD UP TO 1.0----"
390 PRINT "          TRY ENTERING THE ROW AGAIN."
400 GOTO 310
410 PRINT
420 NEXT I

```

```

430 PRINT "DO YOU WANT TO OBSERVE EACH"
440 PRINT "PERIOD UNDER ANALYSIS (Y/N)";
450 INPUT A$
460 IF A$="Y" THEN 480
470 IF A$<>"N" THEN 430
480 REM LOOP TO MULTIPLY VECTOR (V1) BY TRANSITION MATRIX (T)
490 N=N+1
500 FOR I=1 TO S
510 V2(I)=0
520 FOR J=1 TO S
530 REM ADD MULTIPLIED COLUMNS TO V2 ARRAY
540 V2(I)=V2(I)+FNR(V1(J)*T(J,I))
550 NEXT J
560 NEXT I
570 REM SKIP PRINTING VECTOR IF NOT REQUESTED
580 IF A$<>"Y" THEN 620
590 PRINT
600 PRINT "POPULATION PROPORTION "
610 PRINT "VECTOR AT PERIOD ";N;"IS:"
620 N1=0
630 FOR I=1 TO S
640 IF A$<>"Y" THEN 660
650 PRINT V2(I)
660 IF V2(I)<>V1(I) THEN 680
670 N1=N1+1
680 V1(I)=V2(I)
690 NEXT I
700 IF N1<> S THEN 480
710 REM PRINT EQUILIBRIUM VECTOR VALUES
720 PRINT
730 PRINT "EQUILIBRIUM REACHED AT PERIOD";N
740 PRINT "VECTOR AT EQUILIBRIUM:"
750 FOR I=1 TO S
760 PRINT FNR(V1(I))
770 NEXT I
780 PRINT
800 REM PRINT TRANSITIONS NEEDED FOR EACH STATE TO BE REOCCUPIED
810 FOR I=1 TO S
820 IF T(I,I)=1 THEN 860
830 IF V1(I)<=0 THEN 860
840 PRINT "1ST PASSAGE--STATE ";I;" ";
850 PRINT FNR(1/V1(I))
860 NEXT I
870 PRINT "DO YOU WANT TO RE-RUN THIS"
880 PRINT "PROGRAM WITH DIFFERENT DATA (Y/N)";
890 INPUT A$
900 IF A$="Y" THEN 80
910 IF A$<>"N" THEN 870
920 END

```



## Option

If you plan on entering large matrices, or if you want to run this program repeatedly with the same data, you should use this option. The program will read input from DATA statements, rather than asking you to enter the population proportion vector and the transition probabilities matrix. Replace lines 170 through 350 with the lines shown below. Also delete lines 870 through 910, and leave line 920 where it is. If you plan to re-run the program without entering the population proportion vector, you must delete lines 242 through 249 if they contain DATA statements for a population proportion vector from a previous run.

```
170 REM IF VECTOR UNKNOWN, ASSIGN EQUAL PROBABILITIES TO EACH
    STATE
175 REM IF VECTOR UNKNOWN, YOU MUST DELETE LINES 242-249,
176 REM OR THE DATA WILL BE READ OUT OF SEQUENCE.
180 FOR I=1 TO S
190 V1(I)=FNR(1/S)
200 NEXT I
210 GOTO 280
220 REM LOOP TO READ POPULATION PROPORTIONS
230 PRINT
240 FOR I=1 TO S
241 REM PUT PROPORTION VECTOR ELEMENTS HERE
242 DATA 0,.6,.33,.07,0
250 PRINT "                VECTOR ELEMENT";I;": ";
260 READ V1(I)
265 PRINT V1(I)
270 NEXT I
280 REM READ TRANSITION MATRIX (I BY J ARRAY)
290 PRINT
300 FOR I=1 TO S
310 K=0
320 FOR J=1 TO S
321 REM PUT TRANSITION PROBABILITIES MATRIX HERE
322 DATA 1,0,0,0,0,.38,.45,.17,0,0,.65,.25,0,.1,0
323 DATA .25,0,0,0,.75,0,0,0,0,1
330 PRINT "                ELEMENT IN ROW";I;"COLUMN";J;
340 READ T(I,J)
345 PRINT T(I,J)
350 K=K+T(I,J)
```

## References

- Cabot, A. Victor, and Harnett, Donald L. *An Introduction to Management Science*. Reading, Mass.: Addison-Wesley, 1977.
- Levin, Richard I., and Kirkpatrick, Charles A. *Quantitative Approaches to Management* (3rd ed.). New York: McGraw-Hill, 1975.

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## Nonlinear Breakeven Analysis

---

This program computes the breakeven point of a product using a non-linear method which more closely reflects actual production situations than a linear method. It incorporates a 'learning curve' for both costs and prices. This curve means that each time production or sales double, cumulative average costs or revenue per unit will increase or decrease by the amount of the curves. Zero curve values mean no change occurs. When you enter different curve values for costs and prices, the program indicates the point of maximum gross profit.

To use the program, enter the unit selling price, the selling price learning curve, the variable costs, the variable costs learning curve, and the fixed costs. Variable costs are those which can be directly ascribed to the production of each unit, such as raw materials. Fixed costs, like rent and wages, generally do not vary with each unit produced.

### Example

Acme Widget Supply is considering producing and marketing a new widget. New machines, employee training, and all other overhead costs associated with production of this widget total \$10,000. Each unit produced requires \$5.00 of raw materials, labor, machine depreciation, etc., but they will need proportionally more machines and personnel to produce more widgets, therefore they will use a 5% cost increase learning curve. The marketing department expects the selling price of \$25.00 to decrease on a 5% curve. What is the breakeven point on the new widget? What is the maximum gross profit margin that Acme may realize? What are total costs and total revenue at maximum gross profit?

Answer: Breakeven will occur at 1,663 units. The maximum gross profit margin is 17.182%. Total costs and revenue at maximum gross profit are \$74,134.00 and \$89,514.00, respectively.

### BREAKEVEN ANALYSIS

```
ENTER THE UNIT PRICE? 25
ENTER UNIT PRICE EROSION RATE
(NEGATIVE VALUE MEANS REVENUE DECREASES AS SALES INCREASE)
? -5

ENTER THE AMOUNT OF VARIABLE COSTS PER UNIT? 5
ENTER VARIABLE COSTS LEARNING RATE
(NEGATIVE VALUE MEANS COSTS DECREASE AS PRODUCTION DOUBLES)
? 5

ENTER THE TOTAL AMOUNT OF FIXED COSTS? 10000

BREAKEVEN POINT = 1663 UNITS
TOTAL REVENUE AT BREAKEVEN = $ 24015

MAXIMUM GROSS PROFIT MARGIN AT 6886 UNITS = 17.182 %

TOTAL REVENUE = $ 89514
TOTAL COSTS = $ 74134

TOTAL PROFIT = $ 15380

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA (Y/N)? N
```

## Practice Problems

1. The selling price is \$30.00, and revenue will decrease by 2.5% each time production doubles. Variable costs are \$1.20 per unit, but cumulative average costs will increase by 8% when production quantities double. Fixed costs are \$180,000.00. What is the breakeven point? What is the maximum gross profit margin?

Answer: Breakeven at 9,945 units, maximum gross profit margin of 71.185% occurs at 246,752 units.

2. With a unit price of \$19.95, variable costs of \$4.75, and fixed costs of \$6800, how many units must be sold to break even? (No price or cost changes will occur; use curve values of zero for both revenue and costs.)

Answer: Breakeven at 447 units.

## Program Listing

```
10 PRINT "BREAKEVEN ANALYSIS"
20 REM -- THESE FUNCTIONS COMPUTE THE CURVATURE
30 DEF FNA(X)=-LOG(1+(X/100))/LOG(2)
40 DEF FNB(X)=LOG(1+(X/100))/LOG(2)+1
50 DEF FNC(X)=INT(((T1-T2)/T1)*1E5+.5)/1000
60 PRINT
70 PRINT "ENTER THE UNIT PRICE";
80 INPUT U
90 PRINT "ENTER UNIT PRICE EROSION RATE"
100 PRINT "(NEGATIVE VALUE MEANS REVENUE DECREASES AS SALES
    INCREASE)"
110 INPUT L1
120 A1=FNA(L1)
130 B1=FNB(L1)
140 PRINT
150 PRINT "ENTER THE AMOUNT OF VARIABLE COSTS PER UNIT";
160 INPUT V
170 PRINT "ENTER VARIABLE COSTS LEARNING RATE"
180 PRINT "(NEGATIVE VALUE MEANS COSTS DECREASE AS PRODUCTION
    DOUBLES)"
190 INPUT L2
200 A2=FNA(L2)
210 B2=FNB(L2)
220 PRINT
230 PRINT "ENTER THE TOTAL AMOUNT OF FIXED COSTS";
240 INPUT F
250 PRINT
260 REM INITIALIZE LAST GUESS, LOW GUESS, HIGH GUESS
270 C=0
280 L=1
290 H=1E4
300 REM CALCULATE POINT USING BINARY SEARCH
```

```

310 B=INT((L+H)/2)
320 REM IF NEW POINT = LAST GUESS, EXIT
330 IF B=C THEN 480
340 REM SET LAST GUESS TO NEW POINT
350 C=B
360 REM CALCULATE TOTAL REVENUE AND TOTAL COSTS AT QUANTITY B
370 T1=INT((U*B^B1)+.5)
380 T2=INT((V*B^B2+F)+.5)
390 REM BREAKEVEN POINT FOUND IF TOTAL REVENUE = TOTAL COSTS
400 IF T1=T2 THEN 480
410 REM ADJUST GUESS HIGH OR LOW POINTS, TRY AGAIN
420 IF T1>T2 THEN 450
430 L=B
440 GOTO 310
450 H=B
460 GOTO 310
470 REM BREAKEVEN POINT FOUND, OUTPUT RESULT
480 PRINT "BREAKEVEN POINT =";B;"UNITS"
490 PRINT "TOTAL REVENUE AT BREAKEVEN = $";T1
500 REM USE THIS SECTION IF FIGURES ARE LINEAR
510 IF L1<>L2 THEN 570
520 PRINT "COSTS AND REVENUE ARE LINEAR."
530 PRINT "NO MAXIMUM GROSS PROFIT MARGIN POSSIBLE."
540 GOTO 680
550 REM OUTPUT MAXIMUM GROSS PROFIT MARGIN DATA FOR NON-LINEAR
    VALUES
560 REM (SKIP THIS SECTION IF FIGURES ARE LINEAR)
570 B=INT((EXP((F*(A1-1))/(V*(A2-A1)))/(1-A2))+.5)
580 T1=INT(U*B^B1)
590 T2=INT(V*B^B2+F)
600 PRINT
610 PRINT "MAXIMUM GROSS PROFIT MARGIN AT";
620 PRINT B;"UNITS =";FNC((T1-T2)/T1);"%"
630 PRINT
640 PRINT "                                TOTAL REVENUE = $";T1
650 PRINT "                                TOTAL COSTS = $";T2
660 PRINT
670 PRINT "                                TOTAL PROFIT = $";T1-T2
680 PRINT
690 PRINT "WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA
    (Y/N)";
700 INPUT Z$
710 IF Z$="Y" THEN 60
720 IF Z$<>"N" THEN 680
730 END

```

## References

- Solomon and Pringle. *An Introduction to Financial Management*. Santa Monica, Calif.: Goodyear Publishing Company, 1977.
- Texas Instruments. *Programable 58/59 Calculator Business Decisions Library* (manual), Part number 1014984-9.

---

## Payoff Matrix Analysis

---

This program evaluates a set of alternatives, each of which has some measurable benefit, or “payoff,” subject to varying states of nature. Under different conditions, payoff amounts could be large or they could become losses. To analyze payoffs in conditions of uncertainty, this program employs three criteria: “maximax” (find the alternative with the highest possible payoff), “maximin” (the best alternative under the worst case), and “minimax regret” (the alternative which minimizes opportunity cost).

To use this program, you should carefully consider your alternatives. They must relate to one another (e.g., you have \$20,000 and you want to know which of 4 types of investments is optimal to make given varying states of the economy). You must be able to ‘guesstimate’ what the payoffs will be (positive, negative or zero) for each alternative under each state of nature, as well as the probability of each state of nature’s occurrence.

The computer will ask you how many states of nature to consider and how many alternatives exist. Then you will enter the payoff matrix row by row, starting with action 1 under state 1, action 2 under state 2, and so on. After you enter the matrix, you will input the probabilities of each state of nature. These probabilities are mutually exclusive, and they must add up to 1.0. The computer will ask you to reenter them if they do not add up to 1.0.

The program shows you what choices are best under the maximax and maximin rules. The computer will optionally display the regret matrix. The optimal maximin regret choice displays, followed by the expected payoff values of each alternative.

### Program Notes

The program allows for 10 states of nature and 10 alternatives. You can change this by modifying line 20 of this program as follows:

```
20 DIM S(N,A), M(A), R(N), X(A)
```

Replace the expression N with the maximum states of nature, and A with the maximum number of alternatives.

### Example

Fred wants to invest capital on the market. He sees his choices as stocks, Baa bonds or options. These three choices will pay off relative to how the economy behaves:

Investment	State of Economy		
	Recession	Stable	Inflation
Stocks	-20	65	200
Baa Bonds	0	80	80
Options	-300	0	300
Probability	.3	.2	.5

How does Fred run the program?

Answer:

### PAYOFF MATRIX ANALYSIS

HOW MANY STATES OF NATURE? 3  
HOW MANY POSSIBLE ACTIONS? 3

PAYOFF OF ACTION 1 IN STATE 1 ? -20  
PAYOFF OF ACTION 1 IN STATE 2 ? 65  
PAYOFF OF ACTION 1 IN STATE 3 ? 200

PAYOFF OF ACTION 2 IN STATE 1 ? 0  
PAYOFF OF ACTION 2 IN STATE 2 ? 80  
PAYOFF OF ACTION 2 IN STATE 3 ? 80

PAYOFF OF ACTION 3 IN STATE 1 ? -300  
PAYOFF OF ACTION 3 IN STATE 2 ? 0  
PAYOFF OF ACTION 3 IN STATE 3 ? 300

ENTER PROBABILITY FOR STATE 1 ? .3  
ENTER PROBABILITY FOR STATE 2 ? .2  
ENTER PROBABILITY FOR STATE 3 ? .5

MAXIMAX PAYOFF OF 300 FROM ACTION 3

MAXIMIN PAYOFF OF 0 FROM ACTION 2

DO YOU WANT TO SEE THE REGRET TABLE (Y/N)? Y

STATE	1	2	3	
ACTION 1	20	15	100	MAX REGRET= 100
ACTION 2	0	0	220	MAX REGRET= 220
ACTION 3	300	80	0	MAX REGRET= 300

MINIMAX REGRET PAYOFF OF 100 FROM ACTION 1

EXPECTED VALUES ARE:

FOR ACTION 1 : 107  
FOR ACTION 2 : 56  
FOR ACTION 3 : 60

DO YOU WANT TO RUN THIS PROGRAM  
AGAIN WITH DIFFERENT DATA (Y/N)? N

## Practice Problems

1. A business is considering a service agreement for its computer system. The service agreement costs \$100 per month, and covers all repairs. Because the system is five years old, it may be necessary to repair it more often than in the past. Downtime for this system can be for minor or major repairs; the minor repairs averaging \$140, and major repairs averaging \$900. The probability of downtime requiring minor repair is .07; for major repairs, .08. What are the payoffs?

Answer: Maximax payoff (cost, in this problem): \$0. Maximin payoff: -\$100. Minimax Regret: \$100. Expected value (cost) of service agreement: \$100. Expected cost of no service agreement: \$81.80.

2. A market researcher is interested in gathering responses to an opinion poll in one day. The researcher is paid for each completed survey. The number of responses depends on the weather, as shown below:

Location	Prevailing Weather		
	Sunny	Cloudy	Rainy
Beach	150	30	0
Door-to-Door	40	70	90
Flea Market	80	50	5
Probability of weather:	.5	.3	.2

What are the optimal alternatives under each criterion?

Answer: Under maximax, option 1 with a payoff of 150; under maximin, option 2 with a payoff of 40; under minimax regret, option 3 with a maximum payoff of 85. Expected values: alternative 1, 81; alternative 2, 61; alternative 3, 43.5.

## Program Listing

```
10 REM ANALYSIS OF A PAYOFF MATRIX 4/7/80 3:00 PM
20 DIM S(10,10),M(10),R(10),X(10)
30 PRINT "PAYOFF MATRIX ANALYSIS"
40 PRINT
50 PRINT "HOW MANY STATES OF NATURE";
60 INPUT N
70 PRINT "HOW MANY POSSIBLE ACTIONS";
80 INPUT A
90 PRINT
100 FOR Q=1 TO A
110 M(Q)=-9E9
120 PRINT
130 FOR P=1 TO N
140 PRINT "PAYOFF OF ACTION";Q;"IN STATE";P;
150 INPUT S(Q,P)
160 NEXT P
170 NEXT Q
180 REM ENTER PROBABILITIES FOR EACH STATE OF NATURE
190 A1=0
200 PRINT
```

```

210 FOR Q=1 TO N
220 PRINT "ENTER PROBABILITY FOR STATE";Q;
230 INPUT P1(Q)
240 A1=A1+P1(Q)
250 R(Q)=0
260 NEXT Q
270 IF A1=1 THEN 330
280 PRINT
290 PRINT "-----PROBABILITIES DO NOT ADD TO 1.0-----"
300 PRINT "          CHECK YOUR ENTRIES AND RE-TRY."
310 PRINT
320 GOTO 190
330 REM CALCULATE MAXIMAX & MAXIMIN VALUES
340 A1=-9E9
350 FOR Q=1 TO A
360 A3=0
370 REM REPLACE A3 WITH THE HIGHEST PAYOFF
380 FOR P=1 TO N
390 IF A1=-9E9 THEN 410
400 IF S(Q,P)<=A1 THEN 430
410 A1=S(Q,P)
420 A2=Q
430 REM PUT MINIMUM PAYOFF OF EACH ACTION IN M()
440 IF M(Q)=-9E9 THEN 460
450 IF S(Q,P)>=M(Q) THEN 470
460 M(Q)=S(Q,P)
470 REM SAVE HIGHEST PAYOFF FOR REGRET TABLE
480 IF S(Q,P)<=R(P) THEN 500
490 R(P)=S(Q,P)
500 NEXT P
510 NEXT Q
520 PRINT
530 PRINT "MAXIMAX PAYOFF OF";A1;"FROM ACTION ";A2
540 PRINT
550 A1=-9E9
560 FOR Q=1 TO A
570 IF M(Q)<A1 THEN 600
580 A1=M(Q)
590 A2=Q
600 NEXT Q
610 PRINT "MAXIMIN PAYOFF OF";A1;"FROM ACTION ";A2
620 PRINT
630 PRINT "DO YOU WANT TO SEE THE REGRET TABLE (Y/N)";
640 INPUT A$
650 IF A$="N" THEN 870
660 IF A$<>"Y" THEN 630
670 PRINT
680 PRINT "STATE"; TAB(10);
690 A1=0
700 REM PRINT HEADINGS FOR TABLE
710 FOR P=1 TO N
720 PRINT P,
730 NEXT P
740 PRINT
750 PRINT

```



```

760 FOR Q=1 TO A
770 PRINT "ACTION";Q;TAB(10);
780 REM PRINT REGRET VALUES
790 A1=0
800 FOR P=1 TO N
810 PRINT R(P)-S(Q,P),
820 IF R(P)-S(Q,P)<=A1 THEN 840
830 A1=R(P)-S(Q,P)
840 NEXT P
850 PRINT "MAX REGRET=";A1
860 NEXT Q
870 FOR Q=1 TO A
880 A1=0
890 FOR P=1 TO N
900 IF R(P)-S(Q,P)<=A1 THEN 930
910 A1=R(P)-S(Q,P)
920 X(Q)=R(P)-S(Q,P)
930 NEXT P
940 NEXT Q
950 A1=0
960 FOR P=1 TO A
970 IF P=1 THEN 990
980 IF X(P)>A1 THEN 1010
990 A1=X(P)
1000 A2=P
1010 NEXT P
1020 PRINT
1030 PRINT "MINIMAX REGRET PAYOFF OF";A1;"FROM ACTION";A2
1040 PRINT
1050 PRINT "EXPECTED VALUES ARE:"
1060 FOR P=1 TO A
1070 A1=0
1080 FOR Q=1 TO N
1090 A1=A1+(S(P,Q)*P1(Q))
1100 NEXT Q
1110 PRINT "FOR ACTION";P;"=";A1
1120 NEXT P
1130 PRINT
1140 PRINT "DO YOU WANT TO RUN THIS PROGRAM"
1150 PRINT "AGAIN WITH DIFFERENT DATA (Y/N)";
1160 INPUT A$
1170 IF A$="Y" THEN 40
1180 IF A$<>"N" THEN 1130
1190 END

```

## Reference

Cabot, A. Victor, and Harnett, Donald L. *An Introduction to Management Science*. Reading, Mass.: Addison-Wesley, 1977.

---

## Bayesian Decision Analysis

---

This program revises probabilities (given multiple states of nature) according to Bayes' Theorem for conditional events, and further evaluates possible actions by use of a payoff matrix. This technique applies to sampling for quality based on subjective probabilities you enter.

To use this program, first enter how many possible states of nature there are; for example, an outgoing lot of products can have three possible outcomes: 99 percent good, 90 percent good, or 85 percent good. Then enter the number of conditional actions; e.g., send out the lot, send out the lot and retool machines to correct defects, or rework the lot and retool the machines. The next set of entries is the payoff matrix. You enter payoffs (or costs as negative numbers) for each action, within each state of nature. Next, enter two probabilities for each state of nature; first, the "prior" probability that each state of nature occurs, and then the "conditional" probability based on the occurrence of that state.

To illustrate, consider the three possibilities above: 99 percent, 90, and 85 percent good. These are conditional probabilities; in other words, "99 percent good" is a possible outcome of a production run. Therefore, if "99 percent good" is the present state of nature, then the probability of 99 percent is conditional based on being in that state of nature. The "prior" probability is the likelihood of that state of nature's occurrence in the first place. Prior probabilities are often "guesstimates" made by production personnel, based on experience.

The last two entries are the size of the sample in question and the actual number of "successes" in the sample taken. In the example above, you may have looked at 50 pieces out of an outgoing lot of 1000, and you find that five of them are defective. Enter 50 as the sample size, and five as the actual number of successes. The program then prints the expected values of each action, based on revised probabilities. You choose the optimal action from these values, which is usually that action which minimizes costs or maximizes payoff.

After the expected values, the prior probabilities, likelihoods, joint and posterior probabilities print for each action. A final figure, the marginal probability, prints. This is the "unconditional" or expected success rate. You can go back and re-enter a new sample size (or enter zero to end the program).

### Example

The quality control department at Fergis Bolt International estimates that bolts produced fall into three categories; 99 percent acceptable, 90 percent acceptable, and 80 percent acceptable. These three levels of quality occur 70, 20, and 10 percent of the time, respectively. Roland Fergis II wants to impress his father with a comprehensive study which documents how much the company may lose by not making the right quality control decision. He puts together a payoff matrix which looks like this:

Payoffs Actions	if 99% good	if 90% good	if 80% good
Send lot out	-1200	-1800	-2400
Retool machines without rework	-1400	-1600	-2200
Retool machines and rework	-2000	-2000	-2000

The cost of producing the lot itself is \$1200. If the lot is sent out and the quality is less than 99 percent, Fergis will incur costs of returned merchandise. If they decide to retool the machines only, they will incur downtime, but the rate of returned merchandise will be lower for future lots. If the machines are retooled and the bolts are reworked, the lot will be 99 percent good no matter what; therefore, the cost remains constant. How would Roland Jr. run this program? What will be the optimal strategy based payoffs if 46 of 50 bolts sampled are acceptable?

Answer: The optimal strategy is to retool the machines, at an expected cost of \$1,616.75. This sample has a 94.8% probability of being 90 percent free of defects.

#### BAYESIAN DECISION ANALYSIS

HOW MANY STATES OF NATURE? 3  
HOW MANY CONDITIONAL ACTIONS? 3

ENTER PAYOFFS FOR:  
ACTION 1 UNDER STATE 1 ?-1200  
ACTION 1 UNDER STATE 2 ?-1800  
ACTION 1 UNDER STATE 3 ?-2400  
ACTION 2 UNDER STATE 1 ?-1400  
ACTION 2 UNDER STATE 2 ?-1600  
ACTION 2 UNDER STATE 3 ?-2200  
ACTION 3 UNDER STATE 1 ?-2000  
ACTION 3 UNDER STATE 2 ?-2000  
ACTION 3 UNDER STATE 3 ?-2000

ENTER PRIOR AND CONDITIONAL PROBABILITIES:  
FOR STATE 1 ? .7 , .99  
FOR STATE 2 ? .2 , .9  
FOR STATE 3 ? .1 , .8  
ENTER SAMPLE SIZE (0 TO END)? 50

ENTER ACTUAL NUMBER OF SUCCESSES? 46

GIVEN 46 SUCCESSES IN A SAMPLE OF 50 ,  
THE EXPECTED VALUES ARE:  
ACTION 1 :-1809.42408377  
ACTION 2 :-1616.7539267  
ACTION 3 :-2000

PROBABILITY REVISIONS:  

STATE	PRIOR	LIKELIHOOD	JOINT	POSTERIOR
1	.7	.001	.0007	.018
2	.2	.181	.0362	.948
3	.1	.013	.0013	.034

ENTER SAMPLE SIZE (0 TO END)? 0

## Practice Problems

1. In the example above, is the minimum number of acceptable bolts allowable in order to send the lot out without retooling machines? At this point, what is the probability that this lot actually is 99 percent free of defects? (Hint: Find the answer by trial-and-error. Enter a successively smaller number of successes until you get the answer.)

Answer: The minimum is 48 out of 50, with an expected cost of \$1,357.59. At this rate, it is 77.2% likely that the bolts are 99 percent free of defects.

2. In the example above, does action 3 — rework the lot and retool the machines — become optimal?

Answer: At 41 acceptable items from a sample of 50, the cost of \$2,000 is less than the other two alternatives (send out lot: \$2,203.96, send out and retool: \$2,003.96). At this point, it is 67.03% probable that the lot is 80 percent good.

## Program Listing

```
1 PRINT " BAYESIAN DECISION ANALYSIS"
2 PRINT
10 DIM P1(4),P2(4),P3(4),P5(4),A(4,4),M(3)
15 DEF FNR(Z1)=INT(Z1*1000+.5)/1000
20 PRINT "   HOW MANY STATES OF NATURE";
30 INPUT N1
40 PRINT "HOW MANY CONDITIONAL ACTIONS";
50 INPUT A1
60 PRINT
70 PRINT " ENTER PAYOFFS FOR:"
79 REM ENTER PAYOFF MATRIX
80 FOR I=1 TO A1
90 FOR J=1 TO N1
100 PRINT "ACTION";I;"UNDER STATE";J;
110 INPUT A(I,J)
120 NEXT J
130 NEXT I
140 PRINT
149 X0=0
150 PRINT "ENTER PRIOR AND CONDITIONAL PROBABILITIES:"
160 FOR I=1 TO N1
165 PRINT "FOR STATE";I;
170 INPUT P1(I),P2(I)
180 X0=X0+P1(I)
185 P3(I)=0
190 NEXT I
200 IF X0=1 THEN 230
210 PRINT "PRIOR PROBABILITIES DO NOT EQUAL 1.0."
220 GOTO 140
230 PRINT
240 PRINT "ENTER SAMPLE SIZE (0 TO END)";
250 INPUT S
255 IF S=0 THEN 670
260 PRINT
290 PRINT "ENTER ACTUAL NUMBER OF SUCCESSES";
300 INPUT I1
301 REM CALCULATE EXPECTED COST FOR SAMPLE SIZE
320 M(1)=S
330 M(2)=I1
340 M(3)=S-I1
```

```

350 FOR J=1 TO 3
360 IF M(J)=0 THEN 420
370 Z=1
380 FOR K=1 TO M(J)
390 Z=Z*K
400 NEXT K
410 M(J)=LOG(Z)
420 NEXT J
430 P4=0
450 FOR H=1 TO N1
459 REM STORE LIKELIHOOD IN P5()
460 P5(H)=FNR(EXP(M(1)-M(2)-M(3)+I1*LOG(P2(H))+(S-I1)*LOG(1-P2(H))))
469 REM STORE JOINT PROBABILITY IN P3()
470 P3(H)=P5(H)*P1(H)
474 REM SUM JOINT PROBABILITIES IN P3()
475 P4=P4+P3(H)
480 NEXT H
489 REM CALCULATE EXPECTED MONETARY VALUES
490 FOR I=1 TO A1
500 E(I)=0
510 FOR J=1 TO N1
520 E(I)=E(I)+(A(I,J)*(P3(J)/P4))
530 NEXT J
535 NEXT I
540 PRINT
550 PRINT "GIVEN";I1;"SUCCESSSES IN A SAMPLE OF";S;"",
560 PRINT "THE EXPECTED VALUES ARE:"
570 FOR I=1 TO A1
580 PRINT "ACTION";I;":":E(I)
590 NEXT I
600 PRINT
610 PRINT "PROBABILITY REVISIONS:"
620 PRINT "STATE PRIOR LIKELIHOOD JOINT POSTERIOR"
630 FOR I=1 TO N1
640 PRINT I;TAB(8);P1(I);TAB(16);P5(I);TAB(28);P3(I);
645 PRINT TAB(36);FNR(P3(I)/P4)
650 NEXT I
660 GOTO 240
670 END

```

## References

Cabot and Harnett. *An Introduction to Management Science*. Reading, Mass.: Addison-Wesley, 1977.

---

## Economic Order Quantity

---

The purpose of this program is to determine the economic order quantity of an item. You must enter the number of available price breaks, minimum and maximum quantities and unit price for each level, the inventory holding cost as a percentage of each unit's cost, cost of placing an order (in dollars), and the annual demand quantity. The program will compute the EOQ of each price break and indicate if that quantity is within the minimum and maximum quantities for that level.

### Program Notes

It may be more convenient for you to enter holding costs as a fixed dollar amount per unit. Make these changes:

```
150 PRINT "  ENTER THE UNIT HOLDING COST ($)";  
  .  
  .  
  .  
200 H=H/100                                     (DELETE THIS LINE)  
  .  
  .  
  .  
310 E=INT(SQR((2*D*S)/H))
```

Your price breaks may be computed as a percentage discount from a fixed price. Make these changes:

```
60 PRINT "ENTER THE NUMBER OF AVAILABLE PRICE BREAKS";  
70 INPUT B  
72 PRINT "ENTER THE BASE UNIT PRICE";  
74 INPUT U1  
80 PRINT  
90 PRINT "ENTER MINIMUM QUANTITY, MAXIMUM QUANTITY, DISCOUNT"  
100 FOR I=1 TO B  
110 PRINT "AT PRICE BREAK"; I;  
120 INPUT Q(1,I),Q(2,I),D1  
122 U(I)=U1-INT(U1*D1+.5)/100  
130 NEXT I  
140 PRINT
```

### Example

Joe Blow, purchasing agent for a small manufacturer, needs to order motor armatures from a machine shop. The machine shop offers three price breaks to Joe's company: 0 to 499 units, \$5.00 per unit; 500 to 999, \$4.50 per unit; 1000 and up, \$3.90 per unit. Joe's company requires 10,000 units each year. \$20.00 in clerks' time and forms is needed to place an order. About 20% of each unit's cost is spent on warehousing, shipping, breakage, etc. How many orders of how many units should be placed this year in order to minimize costs?

Answer: Joe should place 15 orders of 666 units each.

## ECONOMIC ORDER QUANTITY

ENTER THE NUMBER OF AVAILABLE PRICE BREAKS? 3

ENTER MINIMUM QUANTITY, MAXIMUM QUANTITY, PRICE

AT PRICE BREAK 1 ? 0,499,5

AT PRICE BREAK 2 ? 500,999,4.5

AT PRICE BREAK 3 ? 1000,99999,3.9

ENTER THE UNIT HOLDING COST (% PER UNIT)? 20

ENTER THE COST OF PLACING AN ORDER (\$)? 20

ENTER THE DEMAND QUANTITY PER YEAR (0=END)? 10000

EOQ	# OF ORDERS	QUANTITIES	UNIT PRICE
632	16	0 - 499	5 -- NOT POSSIBLE
666	15	500 - 999	4.5
716	14	1000 - 99999	3.9 -- NOT POSSIBLE

ENTER THE DEMAND QUANTITY PER YEAR (0=END)? 0

WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA? (Y/N)? N

### Practice Problems

1. Three price breaks: \$2.50 per unit for 0-999 units; \$2.25 each for 1,000-1,999 units; 2,000-9,999 units cost \$2.00 each. Cost of placing an order is \$50.00, and holding costs represent 10% of an item's cost. What is the EOQ if annual demand is 5,065 units?

Answer: EOQ is 4 orders of 1,500 units each.

2. Four price breaks: \$89.00 each for 0-9 units; \$82.50 per unit for 10 to 19 units; 20 to 29 units are \$78.00 each; 30 and up are \$75.00 apiece. Cost of placing an order is \$75.00. Holding costs are 15%. What is the EOQ if annual use is 50 units?

Answer: The EOQ is 2 orders of 25 units each.

### Program Listing

```
10 PRINT "ECONOMIC ORDER QUANTITY"
20 REM -- CHANGE SIZE OF ARRAYS Q(2,N) AND U(N) AS NECESSARY
30 REM -- WHERE N = MAXIMUM NUMBER OF PRICE BREAKS YOU WILL USE
40 DIM Q(2,10),U(10)
50 PRINT
60 PRINT "ENTER THE NUMBER OF AVAILABLE PRICE BREAKS";
70 INPUT B
80 PRINT
90 PRINT "ENTER MINIMUM QUANTITY, MAXIMUM QUANTITY, PRICE"
100 FOR I=1 TO B
110 PRINT "AT PRICE BREAK";I;
120 INPUT Q(1,I),Q(2,I),U(I)
130 NEXT I
140 PRINT
150 PRINT "ENTER THE UNIT HOLDING COST (% PER UNIT)";
160 INPUT H
170 IF H>0 THEN 200
180 PRINT "HOLDING COST MUST BE GREATER THAN ZERO."
190 GOTO 150
200 H=H/100
```

```

210 PRINT "    ENTER THE COST OF PLACING AN ORDER ($)";
220 INPUT S
230 PRINT "ENTER THE DEMAND QUANTITY PER YEAR (0=END)";
240 INPUT D
250 IF D=0 THEN 440
260 PRINT
270 REM OUTPUT THE RESULTS
280 PRINT "    EOQ    ", "# OF ORDERS", "QUANTITIES", "UNIT PRICE"
290 REM CALCULATE EOQ BY FORMULA FOR EACH PRICE BREAK
300 FOR I=1 TO B
310 E=INT(SQR((2*D*S)/(U(I)*H)))
320 PRINT E, INT(D/E+.9), Q(1,I); "--"; Q(2,I), U(I),
330 REM TEST TO SEE IF  EOQ  FALLS WITHIN ORDER QUANTITY FOR THIS
    PRICE
340 IF Q(1,I)>E THEN 390
350 IF Q(2,I)<E THEN 390
360 PRINT
370 GOTO 400
380 REM PRICE BREAK IS NOT AVAILABLE AT THIS EOQ
390 PRINT "-- NOT POSSIBLE"
400 NEXT I
410 PRINT
420 GOTO 230
430 REM RESTART OR END PROGRAM?
440 PRINT
450 PRINT "WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA?
    (Y/N)";
460 INPUT Z$
470 IF Z$="Y" THEN 50
480 IF Z$<>"N" THEN 450
490 END

```

## References

- Chase and Aquilano. *Production and Operations Management*. Homewood, Ill.: Richard D. Irwin, Inc., 1977.
- McLaughlin and Pickhardt. *Quantitative Techniques for Management Decisions*. New York: McGraw-Hill, 1975.



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## Economic Production Quantity

---

It is often useful to know the optimal quantity of an item to produce in order to minimize expenses. This program computes that quantity for a given item, and incorporates simultaneous sales calculations (where units are being sold while more are being produced).

To use the program, enter the rate of production, the sales or use rate (the average number of units removed from inventory each day), the total number of units sold in a year, the holding cost (in dollars per unit), and the setup cost. The program will output the optimal number of setups per year, and the optimum quantity to produce in each lot. The optimum quantity is that which minimizes setup and carrying costs.

### Example

Waldo's Paint Factory produces several different paint colors using a single mixing and filling machine. The machine will produce 300 gallons each day, and currently Waldo ships 125 gallons of each color every day, and 35,000 gallons per year. Holding costs are \$0.15 per gallon. For each lot produced, the machine must be completely cleaned, at a cost of \$150. How many lots of each color per year should Waldo produce? How many gallons in each lot?

Answer: Each year, Waldo should run 3 lots of 11,666 gallons each.

### ECONOMIC PRODUCTION QUANTITY

```
ENTER THE RATE OF PRODUCTION (UNITS/DAY)? 300
ENTER THE SALES OR USE RATE (UNITS/DAY)? 125
      ENTER ANNUAL SALES OR USE? 35000
ENTER THE UNIT HOLDING COST ($ PER UNIT)? .15
      ENTER THE SETUP COST ($)? 150
```

```
OPTIMAL NUMBER OF SETUPS = 3 PER YEAR
      EPQ = 11666 UNITS
```

```
WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA? (Y/N)? N
```

### Practice Problems

1. Daily production of 45 units, daily sales of 20 units. Annual sales total 4,000 units. Holding costs are \$0.67 per unit. Setup costs are \$25.00. What is the EPQ?

Answer: 5 lots of 800 units each.

2. 50 units per day are produced, 35 are sold. Annually, 6,500 units are sold. Holding costs are \$0.45 per unit. Setup costs are \$60.00 per lot. How many lots are optimum? What size lots?

Answer: 3 lots of 2,166 units each.

## Program Listing

```
10 PRINT "ECONOMIC PRODUCTION QUANTITY"
20 PRINT
30 PRINT "ENTER THE RATE OF PRODUCTION (UNITS/DAY)";
40 INPUT R
50 IF R>0 THEN 100
60 PRINT
70 PRINT "PRODUCTION RATE MUST BE GREATER THAN ZERO."
80 PRINT
90 GOTO 30
100 PRINT " ENTER THE SALES OR USE RATE (UNITS/DAY)";
110 INPUT U
120 IF U>=0 THEN 170
130 PRINT
140 PRINT "SALES (USE) RATE MUST BE NON-ZERO."
150 PRINT
160 GOTO 100
170 PRINT "                ENTER ANNUAL SALES OR USE";
180 INPUT H
190 IF H>=U THEN 240
200 PRINT
210 PRINT "ANNUAL RATE MUST BE HIGHER THAN DAILY RATE"
220 PRINT
230 GOTO 170
240 PRINT "ENTER THE UNIT HOLDING COST ($ PER UNIT)";
250 INPUT J
260 IF J>0 THEN 310
270 PRINT
280 PRINT "HOLDING COST MUST BE GREATER THAN ZERO."
290 PRINT
300 GOTO 240
310 PRINT "                ENTER THE SETUP COST ($)";
320 INPUT S
330 PRINT
340 IF S>0 THEN 380
350 PRINT "SETUP COST MUST BE GREATER THAN ZERO."
360 PRINT
370 GOTO 310
380 REM OUTPUT THE RESULTS
390 N=INT(SQR(((J*H)/(2*S))*(1-(U/R)))+.5)
400 PRINT "OPTIMAL NUMBER OF SETUPS =";N;"PER YEAR"
410 PRINT "                EPQ =";INT(H/N);"UNITS"
420 REM RESTART OR END PROGRAM?
430 PRINT
440 PRINT "WOULD YOU LIKE TO RE-RUN THIS PROGRAM WITH NEW DATA?"
    (Y/N)";
450 INPUT Z$
460 IF Z$="Y" THEN 20
470 IF Z$<>"N" THEN 440
480 END
```

## Reference

McLaughlin and Pickhardt. *Quantitative Techniques for Management Decisions*. New York: McGraw-Hill, 1975.

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## Statistical Estimation Theory

---

Statistical estimation theory is the science of determining unbiased estimates for various statistics from sample figures, establishing confidence interval estimates for those statistics, and determining the number of samples that must be taken to reduce the probability of error in these estimates to stated maxima. This program performs these calculations.

At the start of the program you must enter the size of the sample, the mean of the sample, and the sample variance. The program then prints the unbiased estimate of the population variance and, for both the mean and the standard deviation, each of seven different confidence levels, the confidence interval estimate, and the maximum and minimum values produced thereby. You may then have the program calculate how large a sample you would have to take to reduce the error of your estimate to a given maximum. You enter the desired confidence level, the maximum desired error, and whether you are testing the mean or the standard deviation. The program then calculates the sample size needed.

### Example

A government researcher did a study to determine how long people had to wait in line at the post office. He took 100 samples; the mean of the sample was 15 minutes, and the sample variance was 2.02. At each of the seven confidence levels, what is the maximum and minimum for the mean and standard deviation? How many samples would have to be taken to be 99% confident that the error in the mean was no greater than .2?

Answer:

```
STATISTICAL ESTIMATION THEORY
ENTER NUMBER OF SAMPLES TAKEN
? 100
ENTER MEAN OF SAMPLE
? 15
ENTER SAMPLE VARIANCE
? 2.02
UNBIASED ESTIMATE OF SIGMA SQUARED
POPULATION VARIANCE = 2.04040404
CONFIDENCE INTERVAL ESTIMATES FOR MEAN:
```

CONFIDENCE LEVEL	PLUS OR MINUS	MAXIMUM	MINIMUM
50	.096346016	15.096346	14.903654
60	.120219488	15.1202195	14.8797805
70	.148046977	15.148047	14.851953
80	.183060302	15.1830603	14.8169397
90	.234955361	15.2349554	14.7650446
95	.279966588	15.2799666	14.7200334
99	.367938199	15.3679382	14.6320618

CONFIDENCE INTERVAL ESTIMATES  
FOR STANDARD DEVIATION:

50	.0681269213	1.49655404	1.3603002
60	.0850080148	1.51343514	1.34341911
70	.104685021	1.53311214	1.3237421
80	.129443181	1.5578703	1.29898394
90	.166138529	1.59456565	1.26228859
95	.197966273	1.62639339	1.23046085
99	.260171595	1.68859872	1.16825553

DO YOU WANT A CALCULATION OF HOW LARGE  
A SAMPLE YOU MUST TAKE TO REDUCE  
THE ERROR OF YOUR ESTIMATE TO A  
MAXIMUM QUANTITY? (Y/N)

? Y

ENTER YOUR CHOSEN CONFIDENCE LEVEL  
(FROM ABOVE CHOICES ONLY), 1 FOR 50,  
2 FOR 60, 3 FOR 70, 4 FOR 80, 5 FOR 90,  
6 FOR 95, AND 7 FOR 99

? 7

ENTER MAXIMUM DESIRED ERROR OF ESTIMATE

? .2

ARE YOU TESTING THE MEAN (M) OR THE  
STANDARD DEVIATION (S)?

? M

AT THE 99 PERCENT CONFIDENCE LEVEL  
IT WOULD BE NECESSARY TO TAKE 339  
SAMPLES TO BE SURE THAT YOUR ESTIMATE  
OF THE ERROR IN THE MEAN  
DID NOT EXCEED .2

DO YOU HAVE NO MORE CALCULATIONS (0),  
MORE WITH THE SAME SAMPLES (1), OR  
BRAND-NEW SAMPLING (2)?

? 0

### Practice Problems

1. Using the data from the above example, how many samples would have to be taken to reduce the error in the standard deviation to .075 at the 99% confidence level?

Answer: 1,204

2. If all the data is the same as in the above example, how many samples must be taken to reduce the error in the mean to .1 at the 95% confidence level?

Answer: 784

## Program Listing

```
10 PRINT "STATISTICAL ESTIMATION THEORY"
20 DIM C(7),F(7)
29 REM READ CONFIDENCE LEVELS AND COEFFICIENTS
30 FOR I=1 TO 7
40 READ C(I),F(I)
50 NEXT I
60 PRINT "ENTER NUMBER OF SAMPLES TAKEN"
70 INPUT N
80 PRINT "ENTER MEAN OF SAMPLE"
90 INPUT X
100 PRINT "ENTER SAMPLE VARIANCE"
110 INPUT S2
120 S1=S2*N/(N-1)
130 PRINT "UNBIASED ESTIMATE OF SIGMA SQUARED"
140 PRINT "POPULATION VARIANCE =" ; S1
150 S=SQR(S1)
280 S3=S/SQR(N)
290 PRINT "CONFIDENCE INTERVAL ESTIMATES FOR MEAN:"
300 PRINT
310 PRINT "CONFIDENCE    PLUS OR"
320 PRINT "  LEVEL", "    MINUS", "MAXIMUM", "MINIMUM"
330 FOR I=1 TO 7
340 PRINT "    ";C(I),F(I)*S3,X+F(I)*S3,X-F(I)*S3
350 NEXT I
360 PRINT
370 PRINT "CONFIDENCE INTERVAL ESTIMATES"
380 PRINT "FOR STANDARD DEVIATION:"
390 PRINT
400 FOR I=1 TO 7
410 J=F(I)*S/SQR(2*N)
420 PRINT "    ";C(I),J,S+J,S-J
430 NEXT I
440 PRINT
450 PRINT "DO YOU WANT A CALCULATION OF HOW LARGE"
460 PRINT "A SAMPLE YOU MUST TAKE TO REDUCE"
470 PRINT "THE ERROR OF YOUR ESTIMATE TO A"
480 PRINT "MAXIMUM QUANTITY? (Y/N)"
490 INPUT B$
500 IF B$="N" THEN 780
510 IF B$<>"Y" THEN 450
520 PRINT "ENTER YOUR CHOSEN CONFIDENCE LEVEL"
530 PRINT "(FROM ABOVE CHOICES ONLY), 1 FOR 50,"
540 PRINT "2 FOR 60, 3 FOR 70, 4 FOR 80, 5 FOR 90,"
545 PRINT "6 FOR 95, AND 7 FOR 99"
550 INPUT J
560 PRINT "ENTER MAXIMUM DESIRED ERROR OF ESTIMATE"
570 INPUT M
580 PRINT "ARE YOU TESTING THE MEAN (M) OR THE"
590 PRINT "STANDARD DEVIATION (S)?"
600 INPUT C$
```

```

610 IF C#="S" THEN 680
620 IF C#<>"M" THEN 580
640 N3=INT((S*F(J)/M)^2)+1
650 GOTO 690
680 N3=INT(((F(J)*S/M)^2)/2)+1
690 PRINT "AT THE";C(J);"PERCENT CONFIDENCE LEVEL"
700 PRINT "IT WOULD BE NECESSARY TO TAKE";N3
710 PRINT "SAMPLES TO BE SURE THAT YOUR ESTIMATE"
720 PRINT "OF THE ERROR IN THE ";
730 IF C#="S" THEN 760
740 PRINT "MEAN"
750 GO TO 770
760 PRINT "STANDARD DEVIATION"
770 PRINT "DID NOT EXCEED";M
780 PRINT "DO YOU HAVE NO MORE CALCULATIONS (0),"
790 PRINT "MORE WITH THE SAME SAMPLES (1),OR"
800 PRINT "BRAND-NEW SAMPLING (2)?"
810 INPUT Y
820 IF Y=1 THEN 440
830 IF Y=2 THEN 60
900 DATA 50, 0.6744902454373
910 DATA 60, 0.8416214285714
920 DATA 70, 1.0364335334476
930 DATA 80, 1.2815515669516
940 DATA 90, 1.6448536821705
950 DATA 95, 1.9599641025641
960 DATA 99, 2.575827586207
999 END

```

## References

- Harnett. *Introduction to Statistical Methods*. 2d ed. Reading, Mass.: Addison-Wesley, 1975.
- Spiegel. *Statistics*. New York: McGraw-Hill, 1961.

---

## Statistics

---

This program analyzes grouped and ungrouped data which you enter, and prints as many as 26 statistics: measures of central tendency, variance, skewness, kurtosis, and correlation.

When you run the program, enter the total population (if known), or 0 (if unknown). If the data are grouped, enter "G;" if ungrouped, enter "U." The next step is to enter the frequency, followed by the value observed at that frequency. After the last item, enter a frequency and value of 0. If you are entering ungrouped data, just enter the observations; enter 9E9 after the last one. The program then calculates and prints the statistics, indicating which are not available based on the data entered.

### Program Notes

This program accepts a maximum of 250 grouped or ungrouped observations. To change this, modify lines 10 and 15 of the program as follows:

```
10 DIM S(40),X(I),Y(I),Z(I)
15 N1=I
```

Replace the expression I with a constant equal to the maximum number of observations.

### Example

Randy Flashpan is a local disk jockey. His weekly show has a segment where listeners phone in their evaluations of certain songs by rating them on a scale of 1 to 10. 100 listeners called in their scores on one record, and their scores are listed below:

Score	Number of Listeners
1	13
2	6
3	2
4	4
5	10
6	13
7	22
8	18
9	10
10	2

In Randy's lexicon, a song with a median score of 7 or more is "boss hit-bound." If the median is between 5 and 7, the song is classified as "lukewarm." If the median falls below 4, the record is totally dropped from the radio station's playlist.

Based on the sample data shown, how should Randy classify the record? Furthermore, how does someone with the intelligence of a disk jockey run this program?

Answer: This song resides in the lukewarm category, with a median of 6.59.



## STATISTICS

ENTER TOTAL POPULATION (0=UNKNOWN)? 100

ARE DATA (G)GROUPED OR (U) UNGROUPED? G

ENTER FREQUENCY, THEN VALUE (0,0=END)

PAIR NO. 1 ? 13 , 1

PAIR NO. 2 ? 6 , 2

PAIR NO. 3 ? 2 , 3

PAIR NO. 4 ? 4 , 4

PAIR NO. 5 ? 10 , 5

PAIR NO. 6 ? 13 , 6

PAIR NO. 7 ? 22 , 7

PAIR NO. 8 ? 18 , 8

PAIR NO. 9 ? 10 , 9

PAIR NO. 10 ? 2 , 10

PAIR NO. 11 ? 0 , 0

RESULTS TABULATED AS FOLLOWS:

TOTAL POPULATION: 100

DATA ARE: GROUPED

NO. OF SAMPLES: 100

SUM OF SAMPLES: 583

MEAN: 5.83

SUM OF SQUARES: 4077

MEAN DEVIATION: 2.141

MEDIAN: 6.59090909

VARIANCE: 6.78109996

STANDARD DEVIATION: 2.60405452

UNBIASED ESTIM. OF VARIANCE: 6.84959592

STD. DEV. USING THAT VARIANCE: 2.61717327

PROBABLE ERROR: 1.75640874

STD. ERROR OF MEAN: .261717327

COEFF. OF VARIATION: 44.6664584 %

3RD MOMENT ABOUT MEAN: -11.946726

4TH MOMENT ABOUT MEAN: 105.989549

MOMENT COEFF. SKEWNESS: -.676548108

MOMENT COEFF. KURTOSIS: 2.30495658

UNBIASED ESTIM. 3RD CENT. MOMENT: -12.3136735

STD ERR. MEAN WITH FINITE POPULATION

CORRECTION FACTOR: 0

PEARSON'S 2ND COEFF. SKEWNESS: -.876605023

RANGE: 9

INDEX OF MEAN DEV. TO PRODUCT OF

M.A.E. AND STD. DEV.: 1.03044907

## Practice Problems

1. Meter readings from a holding tank at a fuel processing plant are: 12.98, 13.001, 18.25, 4.4, 9.8, 11, 14.5, 12.7, 7.2, and 6.1. What are the mean and median meter readings? What is the standard deviation?

Answer: The mean reading is 10.9931; the median is 11.85. The standard deviation is 3.998.

2. An actuarial clerk wants statistics on the population of Casper County relative to the occurrence of heart disease. The table below shows age brackets and the number of diagnosed heart disease cases for those ages:

Age	Diagnosed Cases (per 1000 people)
0-5	6
6-10	5
11-20	3
21-25	8
26-30	7
31-35	12
36-40	17
41-45	19
46-50	30
51-55	35
56-60	43
61-65	50
66-70	61

What is the median age of the onset of heart disease in Casper County? 1,200 cases were evaluated. What is the measure of skewness for this population, since it appears to be skewed to the right of the mean? What is the standard error of the mean? (Hint: You must increase array sizes on line 10 to 300.)

Answer: The median age is 58 for the onset of heart disease. Skewness -1.261. The standard error of the mean is .90324.

## Program Listing

```
1 PRINT "STATISTICS"
2 PRINT
10 DIM S(40),X(250),Y(250),Z(250)
14 REM N1= DIMENSION OF X, Y & Z
15 N1=250
20 FOR I=1 TO 40
30 S(I)=0
40 NEXT I
45 S1=0
50 PRINT "ENTER TOTAL POPULATION (0=UNKNOWN)";
70 INPUT T9
75 PRINT
80 PRINT "ARE DATA (G)GROUPED OR (U) UNGROUPED";
90 INPUT U$
95 PRINT
100 IF U$="G" THEN 440
104 REM ---- UNGROUPED DATA
105 J=1
110 PRINT "ITEM NO.";J;
119 REM-ENTER 9E9 AFTER LAST ITEM
120 INPUT X(J)
130 IF X(J)<>9E9 THEN 150
140 J=J+1
145 GOTO 190
149 REM-CALCULATES NO. OF ITEMS
```

```

150 S(1)=S(1)+1
159 REM-CALCULATES SUM OF ITEMS
160 S(2)=S(2)+X(J)
169 REM-CALCULATES THE SUM OF SQUARES
170 S(4)=S(4)+X(J)*X(J)
175 J=J+1
180 IF J<N1 THEN 110
189 REM-CALCULATES MEAN
190 S(3)=S(2)/S(1)
209 REM-CALC. DEVIATION FROM MEAN
210 S(5)=ABS(S(3)-X(J))
219 REM-CAL. SUM OF DEVIATIONS
220 S(6)=S(6)+S(5)
229 REM-CALC. 3RD POWER OF DEVIATION
230 S(8)=(X(J)-S(3))^3
239 REM-CALC. SUM OF 3RD POWERS
240 S(9)=S(9)+S(8)
249 REM-CALC. 4TH POWER OF DEVIATION
250 S(10)=(X(J)-S(3))^4
259 REM-CALC. SUM OF 4TH POWERS
260 S(11)=S(11)+S(10)
279 REM-CALC. MEAN DEVIATION
280 S(7)=S(6)/S(1)
288 REM USE SHELL-METZNER SORT TO
289 REM-ARRANGE DATA IN ASCENDING ORDER
290 M1=S(1)
295 M1=INT(M1/2)
300 IF M1=0 THEN 370
305 K=S(1)-M1
310 J=1
315 I=J
320 L=I+M1
325 IF X(I)<=X(L) THEN 355
330 W=X(I)
335 X(I)=X(L)
340 X(L)=W
345 I=I-M1
350 IF I>=M1 THEN 320
355 J=J+1
360 IF J>K THEN 295
365 GOTO 315
369 REM-CALC. MEDIAN
370 IF S(1)/2=INT(S(1)/2) THEN 410
379 REM ODD NO. OF ITEMS
380 M=S(1)/2+.5
390 S(12)=X(M)
400 GOTO 840
409 REM EVEN NO. OF ITEMS
410 M=S(1)/2
420 S(12)=(X(M)+X(M+1))/2
430 GO TO 840
439 REM ----GROUPED DATA----
440 PRINT "ENTER FREQUENCY, THEN VALUE (0,0=END)"
445 J=1

```

```

450 PRINT "PAIR NO.":J:
459 REM-ENTER 0,0 AFTER LAST ITEM
460 INPUT Y(J),Z(J)
470 IF Y(J)=0 THEN 529
489 REM-CALC. NO. OF SAMPLES
490 S(1)=S(1)+Y(J)
495 S1=S1+1
499 REM-CALC. TOTAL OF VALUES
500 S(2)=S(2)+Y(J)*Z(J)
509 REM-CALC. SUM OF SQUARES
510 S(4)=S(4)+Y(J)*Z(J)*Z(J)
520 J=J+1
525 IF J<=N1 THEN 450
529 REM-CALC. MEAN
530 S(3)=S(2)/S(1)
540 FOR J=1 TO S(1)
549 REM-CALC. ABSOLUTE DEVIATION
550 S(5)=Y(J)*ABS(S(3)-Z(J))
559 REM-CALC. SUM OF ABS. DEVIATIONS
560 S(6)=S(6)+S(5)
569 REM-CALC. 3RD POWER OF DEVIATIONS
570 S(8)=Y(J)*(Z(J)-S(3))^3
579 REM-CALC. SUM OF 3RD POWERS
580 S(9)=S(9)+S(8)
589 REM-CALC. 4TH POWERS OF DEVIATIONS
590 S(10)=Y(J)*(Z(J)-S(3))^4
599 REM-CALC. SUM OF 4TH POWERS
600 S(11)=S(11)+S(10)
610 NEXT J
619 REM-CALC. MEAN DEVIATION
620 S(7)=S(6)/S(1)
628 REM USE SHELL-METZNER SORT TO
629 REM-ARRANGE DATA IN ASCENDING ORDER
630 M1=S1
635 M1=INT(M1/2)
640 IF M1=0 THEN 740
645 K=S1-M1
650 J=1
655 I=J
660 L=I+M1
665 IF Z(I)<=Z(L) THEN 710
670 V=Y(I)
675 W=Z(I)
680 Y(I)=Y(L)
685 Z(I)=Z(L)
690 Y(L)=V
695 Z(L)=W
700 I=I-M1
705 IF I>=1 THEN 660
710 J=J+1
715 IF J>K THEN 635
720 GOTO 655
739 REM-CALCULATES MEDIAN
740 T=0
750 K=1

```

```

760 IF T+Y(K)>S(1)/2 THEN 800
765 T=T+Y(K)
770 K=K+1
780 GOTO 760
785 IF K<=S(1) THEN 750
800 S(12)=(Z(K)+Z(K-1))/2+((Z(K)-Z(K-1))/Y(K))*(S(1)/2-T)
840 N=S(1)
850 PRINT "RESULTS TABULATED AS FOLLOWS:"
860 PRINT "TOTAL POPULATION:";
870 IF T9=0 THEN 900
880 PRINT T9
890 GOTO 910
900 PRINT "UNKNOWN/NOT INDICATED"
905 PRINT
910 PRINT "DATA ARE:";
920 IF U$="G" THEN 950
930 PRINT "UNGROUPED"
940 GOTO 960
950 PRINT "GROUPED"
960 PRINT "NO. OF SAMPLES:";S(1)
970 PRINT "SUM OF SAMPLES:";S(2)
980 PRINT "MEAN:";S(3)
990 PRINT "SUM OF SQUARES:";S(4)
1000 PRINT "MEAN DEVIATION:";S(7)
1010 PRINT "MEDIAN:";S(12)
1020 S(13)=S(4)/N-S(3)^2
1030 PRINT "VARIANCE:";S(13)
1040 IF U$="G" THEN 1070
1050 S(14)=S(13)-(1/12)*(Z(2)-Z(1))^2
1060 PRINT "VARIANCE WITH SHEP. CORR.:";S(14)
1070 S(15)=SQR(S(13))
1080 PRINT "STANDARD DEVIATION:";S(15)
1090 IF U$="G" THEN 1120
1100 S(16)=SQR(S(14))
1110 PRINT "STD. DEV. WITH SHEP. CORR.:";S(16)
1120 S(17)=S(13)*N/(N-1)
1130 PRINT "UNBIASED ESTIM. OF VARIANCE:";S(17)
1140 S(18)=SQR(S(17))
1150 PRINT "STD. DEV. USING THAT VARIANCE:";S(18)
1160 S(19)=.67449*S(15)
1170 PRINT "PROBABLE ERROR:";S(19)
1180 S(20)=SQR(S(17)/N)
1190 PRINT "STD. ERROR OF MEAN:";S(20)
1200 S(21)=S(15)/S(3)
1210 PRINT "COEFF. OF VARIATION:";100*S(21);"%"
1220 S(22)=S(9)/N
1230 PRINT "3RD MOMENT ABOUT MEAN:";S(22)
1240 S(23)=S(11)/N
1250 PRINT "4TH MOMENT ABOUT MEAN:";S(23)
1260 IF U$="G" THEN 1300
1270 R=Z(2)-Z(1)
1280 S(24)=S(23)-.5*(R^2)*S(17)+(7/240)*R^4
1290 PRINT "4TH MOMENT W/ SHEP. CORR.:";S(24)
1300 S(25)=S(22)/(S(15)^3)
1310 PRINT "MOMENT COEFF. SKEWNESS:";S(25)
1320 S(26)=S(23)/(S(13)^2)

```

```

1330 PRINT "MOMENT COEFF. KURTOSIS:";S(26)
1340 S(27)=(S(22)*N^2)/((N-1)*(N-2))
1350 PRINT "UNBIASED ESTIM. 3RD CENT. MOMENT:";S(27)
1360 IF T9=0 THEN 1420
1370 IF N<=.05*T9 THEN 1420
1380 S(28)=S(20)*SQR((T9-N)/(T9-1))
1390 PRINT "STD ERR. MEAN WITH FINITE POPULATION"
1400 PRINT "CORRECTION FACTOR:";S(28)
1410 GOTO 1430
1420 PRINT "FINITE POP. CORR. FACTOR N/A"
1430 S(29)=3*(S(3)-S(12))/S(15)
1440 PRINT "PEARSON'S 2ND COEFF. SKEWNESS:";S(29)
1450 IF U$="G" THEN 1480
1460 S(30)=X(N)-X(1)
1470 GOTO 1490
1480 S(30)=Z(S1)-Z(1)
1490 PRINT "RANGE:";S(30)
1500 S(31)=S(7)/(.7978845608*S(15))
1510 PRINT "INDEX OF MEAN DEV. TO PRODUCT OF"
1520 PRINT "M.A.E. AND STD. DEV.:";S(31)
1530 END

```

## References

- Mendenhall, William, et al. *Statistics: A Tool for the Social Sciences*. Belmont, Calif.: Duxbury Press, 1974.
- Spiegel. *Statistics* (Schaum's Series) New York: McGraw-Hill, 1961.

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## Unbiased Estimator of Standard Deviation

---

The concept of an unbiased estimator of the standard deviation is not common among American statisticians. However, according to the Russian mathematician A. A. Sveshnikov, the unbiased estimator of the standard deviation is given by the following formula:

$$\bar{\sigma} = K_N \sqrt{\frac{1}{N-1} \sum_{j=1}^N (x_j - \bar{x})^2} \quad \text{where} \quad K_N = \sqrt{\frac{N-1}{2} \left( \frac{\Gamma\left(\frac{N-1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} \right)}$$

Using this symbolism  $N$  = sample size, it is easily shown that:

for  $N = 2M$  (even sample size),

while for  $N = 2M+1$  (odd sample size),

$$K_N = \sqrt{\frac{N-1}{2}} \left( \frac{\frac{2M-3}{2} \cdot \frac{2M-5}{2} \cdots \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi}}{(M-1)(M-2) \cdots 2 \cdot 1} \right) \quad K_N = \sqrt{\frac{N-1}{2}} \left( \frac{(M-1)(M-2) \cdots 2 \cdot 1}{\frac{2M-1}{2} \cdot \frac{2M-3}{2} \cdots \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi}} \right)$$

To use the program, you must enter the number of samples, and the sum of the squares of the deviations. The program prints out the unbiased estimator of the standard deviation, and asks if you want another calculation.

### Example

In a class of 35 seventh grade students, the sum of the squares of the deviations for their ages is 3.156. What is the unbiased estimator of the standard deviation?

Answer: 0.30691769

```
UNBIASED ESTIMATOR OF STANDARD DEVIATION
THIS PROGRAM CALCULATES THE UNBIASED
ESTIMATOR OF THE STANDARD DEVIATION
WHEN VARIABLE IS NORMALLY DISTRIBUTED
```

```
ENTER THE SUM OF THE SQUARES
OF THE DEVIATIONS
? 3.156
ENTER THE NUMBER OF SAMPLES
? 35
UNBIASED ESTIMATOR OF STANDARD
DEVIATION= .306917689811
ANOTHER CALCULATION? (Y/N)
? N
```

## Practice Problems

1. If 40 samples are randomly distributed and the sum of the squares of their deviations is 9.63, what is the unbiased estimator of the standard deviation?

Answer: 0.500108774

2. In a group of 26 randomly distributed samples, the sum of the squares of the deviations is 34.953. What is the unbiased estimator of the standard deviation?

Answer: 1.1943016

## Program Listing

```
5 PRINT "UNBIASED ESTIMATOR OF STANDARD DEVIATION"
10 PRINT "THIS PROGRAM CALCULATES THE UNBIASED"
20 PRINT "ESTIMATOR OF THE STANDARD DEVIATION"
30 PRINT "WHEN VARIABLE IS NORMALLY DISTRIBUTED"
40 PRINT
50 PRINT "ENTER THE SUM OF THE SQUARES"
60 PRINT "OF THE DEVIATIONS"
70 INPUT S
80 PRINT "ENTER THE NUMBER OF SAMPLES"
90 INPUT N
99 REM COMPUTE K-SUB-N TERM
100 A = SQR((N-1)/2)
110 FOR M = (((N-1)/2)-1) TO 1 STEP -1
120 A = A * M/(M+.5)
130 NEXT M
139 REM SQR(PI)/2 = .8862269255
140 P = .8862269255
150 IF N/2 = INT(N/2) THEN 170
159 REM ODD SAMPLE SIZE
160 P = 1/P
170 PRINT "UNBIASED ESTIMATOR OF STANDARD"
180 PRINT "DEVIATION= "; A * P * SQR(S/(N-1))
190 PRINT "ANOTHER CALCULATION? (Y/N)"
200 INPUT Y$
210 IF Y$ = "Y" THEN 50
220 END
```



## References

- National Bureau of Standards. *Handbook of Mathematical Functions*. Washington, D.C., 1966.
- Sveshnikov, A. A. *Problems in Probability Theory, Mathematical Statistics and Theory of Random Functions*. New York: Dover, 1968.

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## Chi-Square

---

The chi-square test in statistics tests the compatibility of observed frequencies with the expected or theoretical frequencies. For example, suppose we are testing whether a die is fair or biased. We throw the die 60 times, recording the result each time. If the die is fair, we would expect that each of the six sides would come up close to 10 times during the test. But we know that actual events do not always correspond to theoretical expectations. The chi-square test provides the means of determining whether the observed and theoretical results are so divergent that the die cannot be considered fair.

Chi-square is defined as follows:

$$\chi^2 = \sum_{i=1}^K \frac{(O_i - E_i)^2}{E_i}$$

where O represents the observed frequencies and E the expected frequencies. Statisticians have determined what value (the "5% critical value") the chi-square must be below in order that we be 95% positive that two results are compatible. This program tests whether the actual results fall within that level of confidence. It also employs Yates' correction (which some statisticians prefer and some dislike) to test the results. The chi-square formula with Yates' correction is:

$$\chi^2 = \sum_{i=1}^K \left( \frac{|O_i - E_i| - .5}{E_i} \right)^2$$

The program also tests whether the results are too good (below the 95% critical value), which makes clinical workers suspicious of the results.

The program first asks if the expected frequency is a constant. In the above example, each face of the die is expected to appear 10 times, so the answer is "Yes" and you would enter 10 as the constant. You then enter the observed frequencies one-by-one; enter 99999 after the last one. If the expected frequencies are not constant, the program will ask for each set of observed and expected frequencies. After the last entry, enter 99999,1 to end the sequence.

The program will then calculate the chi-square statistics, both with and without Yates' correction, and print them out, indicating the degrees of freedom. It then tests each statistic against the 5% and 95% critical values, and prints out the results.

### Example

Suppose the results of the 60 throws of the die in the above example are as follows:

Face	Expected	Actual
1	10	9
2	10	8
3	10	12
4	10	10
5	10	13
6	10	8

What are the results of the chi-square test for this data? Can the die be considered fair?

Answer: The die can be considered fair.

```

CHI-SQUARE
IS AMOUNT OF EXPECTED FREQUENCY
CONSTANT? (Y/N)
? Y
ENTER CONSTANT EXPECTED FREQUENCY
? 10
ENTER OBSERVED FREQUENCIES ONE BY
ONE AS REQUESTED BELOW
ENTER 99999 TO END
? 9
? 8
? 12
? 10
? 13
? 8
? 99999
CHI SQUARE FOR THESE
OBSERVATIONS= 2.2
FOR 5 DEGREES OF FREEDOM
WITH YATES' CORRECTION, CHI
SQUARE= 1.35
FIVE PERCENT CRITICAL VALUE OF
CHI SQUARE IS 11.071
THEREFORE THE HYPOTHESIS IS NOT
REJECTED AT THE 5% CRITICAL VALUE

```

### Practice Problems

1. A student in a genetics class is performing an experiment to test classical Mendelian theory. That theory predicts that certain biological characteristics should appear in the species under review in the ratios 9:3:3:1. In the 1600 samples which the student takes they appear 904, 297, 302, and 97 times, respectively. Are these results compatible with orthodox Mendelian theory?

Answer: The unadjusted chi-square result is 0.1511, and with Yates' correction that result is 0.1044. The 5% critical value for 3 degrees of freedom is 7.8147, so the results are compatible. However, the 95% critical value is 0.35185, so either with or without Yates' correction, the results are "too good," and the instructor must view the student's experiment with suspicion.

2. A Las Vegas pit boss noticed that a particular roulette wheel seemed to be coming up red more often than black. He kept track of the next 1000 spins; red came up 546 times, and black 454 times. Is the wheel biased?

Answer: The chi-square without Yates' correction is 8.464, and with it is 8.281. The 5% critical value is 3.8415, and the hypothesis is therefore rejected. The pit boss should junk that roulette wheel immediately.

### Program Listing

```

10 PRINT "CHI-SQUARE"
100 PRINT "IS AMOUNT OF EXPECTED FREQUENCY"
110 PRINT "CONSTANT? (Y/N)"
120 INPUT A$
130 IF A$="N" THEN 500
135 IF A$<>"Y" THEN 100
140 PRINT "ENTER CONSTANT EXPECTED FREQUENCY"
150 INPUT Y
299 REM EXPECTED FREQUENCY IS A CONSTANT

```

```

300 PRINT "ENTER OBSERVED FREQUENCIES ONE BY
310 PRINT "ONE AS REQUESTED BELOW"
315 PRINT "ENTER 99999 TO END"
320 INPUT X
330 IF X=99999 THEN 1000
350 N=N+1
370 S=S+(ABS(X-Y)^2)/Y
390 T=T+((ABS(X-Y)-.5)^2)/Y
400 IF A#="N" THEN 520
410 GOTO 320
499 REM EXPECTED FREQUENCY IS NOT A CONSTANT
500 PRINT "ENTER,PAIR BY PAIR, AS REQUESTED, THE"
510 PRINT "OBSERVED, THEN THE EXPECTED, FREQUENCIES"
515 PRINT "ENTER 99999,1 TO END"
520 INPUT X,Y
530 GOTO 330
1000 PRINT "CHI SQUARE FOR THESE"
1010 PRINT "OBSERVATIONS=";S
1020 PRINT "FOR"; N-1; "DEGREES OF FREEDOM"
1030 PRINT "WITH YATES' CORRECTION, CHI"
1040 PRINT "SQUARE=";T
1099 REM BRANCH FOR CALCULATION OF CRITICAL VALUES
1100 IF N>101 THEN 1600
1110 IF N=101 THEN 1500
1120 IF N>31 THEN 1400
1200 FOR I=1 TO N-1
1210 READ C
1220 NEXT I
1230 FOR I=N TO N+29
1240 READ D
1250 NEXT I
1260 GOTO 2500
1400 C=(N-1)*(1-2/(9*(N-1))+1.6449*SQR(2/(9*(N-1))))^3
1410 D=(N-1)*(1-2/(9*(N-1))-1.6449*SQR(2/(9*(N-1))))^3
1420 GOTO 2500
1500 C=124.342
1510 D=77.9295
1520 GOTO 2500
1600 C=.5*(1.6449+SQR(2*(N-1)-1)))^2
1610 D=.5*(SQR(2/(9*(N-1))-1.6449)^2
2500 PRINT "FIVE PERCENT CRITICAL VALUE OF"
2510 PRINT "CHI SQUARE IS"; C
2520 IF T>C THEN 2700
2530 IF S>C THEN 2800
2540 IF S<D THEN 2900
2550 IF T<D THEN 2900
2600 PRINT "THEREFORE THE HYPOTHESIS IS NOT"
2610 PRINT "REJECTED AT THE 5% CRITICAL VALUE"
2620 GOTO 9999
2700 PRINT "THEREFORE THE HYPOTHESIS IS"
2710 PRINT "REJECTED AT THE 5% CRITICAL VALUE"
2720 GOTO 9999
2800 PRINT "WHILE THE UNADJUSTED CHI SQUARE"
2810 PRINT "VALUES ARE UNACCEPTABLE, THOSE WITH"
2820 PRINT "YATES' CORRECTION ARE NOT; THEREFORE"

```

```

2830 PRINT "SAMPLE SIZES SHOULD BE INCREASED OR"
2840 PRINT "SUBSTITUTE MULTINOMIAL DISTRIBUTION"
2850 PRINT "METHODS"
2860 GOTO 9999
2900 PRINT "AGREEMENT IS TOO GOOD AND SHOULD BE"
2910 PRINT "EXAMINED CRITICALLY, BECAUSE EITHER"
2920 PRINT "WITH OR WITHOUT YATES' CORRECTION, THE"
2930 PRINT "CHI SQUARE VALUE IS BELOW THE 95%"
2940 PRINT "CRITICAL VALUE"
5000 DATA 3.8415,5.9915,7.8147,9.4877,11.071,12.592
5010 DATA 14.067,15.507,16.919,18.307,19.675,21.026
5020 DATA 22.362,23.685,24.996,26.296,27.587,28.869
5030 DATA 30.140,31.410,32.671,33.924,35.173,36.415
5040 DATA 37.653,38.885,40.113,41.337,42.557,43.773
5050 DATA .003932,.10259,.35185,.71072,1.1455
5060 DATA 1.635,2.167,2.733,3.325,3.940
5070 DATA 4.575,5.226,5.892,6.571,7.261
5080 DATA 7.962,8.672,9.390,10.117,10.851
5090 DATA 11.591,12.338,13.091,13.848,14.611
5100 DATA 15.379,16.151,16.928,17.708,18.493
9999 END

```

## References

- Hoel. *Introduction to Mathematical Statistics*, 2nd. ed. New York: John Wiley, 1954.
- Spiegel. *Statistics* (Schaum's series). New York: McGraw-Hill, 1961.

---

## Data Forecasting Divergence

---

This program determines the degree to which a forecast diverges from actual data. You enter pairs of actual data and corresponding forecast; after the last data pair, enter 99999,1. The program will then print out the number of pairs of figures, the total error, the total absolute error, the total squared error, the mean error, the mean absolute error (MAE), the mean square error, and the root mean square error.

### Example

A statistical forecaster determined the following data having made the following respective forecasts:

Data	Forecast
1	1
2	2.2
3	2.9
4	3.9
5	5.3
6	6.1
7	7
8	7.9

What are the error statistics for these figures?

Answer:

```
DATA FORECASTING DIVERGENCE
ENTER DATA AND FORECAST
(99999,1 TO END)
? 1,1
? 2,2.2
? 3,2.9
? 4,3.9
? 5,5.3
? 6,6.1
? 7,7
? 8,7.9
? 99999,1
NO. OF PAIRS OF FIGURES= 8
TOTAL ERROR=-.3
TOTAL ABSOLUTE ERROR= .9
TOTAL SQUARED ERROR= .17
MEAN ERROR=-.0375
MEAN ABSOLUTE ERROR= .1125
MEAN SQUARE ERROR= .02125
ROOT MEAN SQUARE ERROR= .145773797
```

## Practice Problems

1. The actual and predicted results in a city council race are as follows:

	Vote %	Poll%
Candidate A	40.3	42.7
Candidate B	22.5	21.4
Candidate C	16.3	18.2
Candidate D	10.5	6.0
Candidate E	7.2	7.4
Candidate F	3.2	4.3

How accurate were the polls?

Answer: Number of pairs = 6; total error = 0; total absolute error = 11.2; total squared error = 32.08; mean error = 0; mean absolute error = 1.866667; mean square error = 5.346667; root mean square error = 2.312286.

2. A new TV weatherman lasted only one week at the station. Following are the actual and predicted temperatures during that week:

	Actual Temperature	Predicted Temperature
Monday	74	49
Tuesday	70	62
Wednesday	58	75
Thursday	60	82
Friday	65	37
Saturday	73	58
Sunday	70	92

What statistics were on the dismissal notice?

Answer: Number of pairs = 7; total error = 15; total absolute error = 137; total squared error = 2955; mean error = 2.14285714; mean absolute error = 19.5714286; mean square error = 422.142858; root mean square error = 20.5461154.

## Program Listing

```
10 PRINT "DATA FORECASTING DIVERGENCE"
20 PRINT "ENTER DATA AND FORECAST"
30 PRINT "(99999, 1 TO END)"
40 INPUT X,Y
50 IF X=99999 THEN 110
60 T1=T1+1
70 T2=T2+X-Y
80 T3=T3+ABS(X-Y)
90 T4=T4+(ABS(X-Y))^2
100 GOTO 40
110 PRINT "NO. OF PAIRS OF FIGURES=";T1
120 PRINT "TOTAL ERROR=";T2
130 PRINT "TOTAL ABSOLUTE ERROR=";T3
140 PRINT "TOTAL SQUARED ERROR=";T4
150 PRINT "MEAN ERROR=";T2/T1
160 PRINT "MEAN ABSOLUTE ERROR=";T3/T1
170 PRINT "MEAN SQUARE ERROR=";T4/T1
180 PRINT "ROOT MEAN SQUARE ERROR=";SQR(T4/T1)
190 END
```

## Reference

Gilchrist. *Statistical Forecasting*. London: John Wiley, 1976.



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## Newtonian Interpolation

---

This program applies to Newton's forward difference formula for interpolation of a given function. Newton's formula is intended to work when the arguments you use in the interpolation commence just below the argument for which you are seeking the tabular value.

You first enter the independent variables on either side of the value for which you want the tabular value interpolated, followed by that value (your desired independent variable). The program then asks for the precision (in decimal places) you want in your answer. This should not exceed the accuracy of either your original data, or your computer's Basic. The program will cease calculating differences when they drop below this level of accuracy.

You then enter the tabular values immediately below and above the desired tabular value. The program prints out the difference between these values, called the first difference. The program asks for additional tabular values, printing out the new difference each time, until the new difference drops below the level of precision you entered earlier. To end the entry of tabular values before this, you enter 99999 as the new tabular value, and the program will branch to the computation of the answer.

### Example

Bill Miller is going to take out a five-year loan at  $4\frac{1}{4}\%$ . He has a table that shows the factors by which he should multiply the principle of the loan to determine the amount of each monthly payment. Unfortunately, the table only gives figures at half-percent intervals. How should Bill use this program to determine the factor at  $4\frac{1}{4}\%$ ?

Interest Rate	Factor
4%	.018416522
$4\frac{1}{2}\%$	.018643019
5%	.018871233
$5\frac{1}{2}\%$	.019101162
6%	.019332801
$6\frac{1}{2}\%$	.019566148
7%	.019801198
$7\frac{1}{2}\%$	.020037949
8%	.020276394

Answer:

```
INTERPOLATION
NEWTON'S FORWARD DIFFERENCE FORMULA
LOWER INDEPENDENT VARIABLE? .04
UPPER INDEPENDENT VARIABLE? .045
DESIRED INDEPENDENT VARIABLE? .0425
PRECISION(IN DECIMAL PLACES)? 9

ENTER TABULAR VALUE AT .04 ? .018416522
ENTER TABULAR VALUE AT .045 ? .018643019

1 ST DIFFERENCE = 2.26496995E-04
ENTER TABULAR VALUE AT .05 ? .018871233

2 ND DIFFERENCE = 1.71700231E-06
ENTER TABULAR VALUE AT .055 ? .019101162
```

3 RD DIFFERENCE =-1.99361239E-09  
INTERPOLATION IS TO THE ORDER OF  
3 RD DIFFERENCES ANSWER = .0185295558

### Practice Problems

1. Jeanne needs to know the sine of 0.63, using the following table. What is that figure?

X	0.6	0.7	0.8	0.9	1.0
SIN X	0.564642	0.644218	0.717356	0.783327	0.841471

Answer: The sine of 0.63 is approximately .589191.

2. Joe Statistics wants to determine the area under the normal curve at .095 standard deviation to the right of the mean. From the following table, what is that area?

Std. Deviations	0.08	0.09	0.1	0.11	0.12
Area	0.53188	0.53586	0.53983	0.54380	0.54776

Answer: The area is .53784.

### Program Listing

```

10 PRINT "          INTERPOLATION"
20 PRINT "NEWTON'S FORWARD DIFFERENCE FORMULA"
30 PRINT " LOWER INDEPENDENT VARIABLE";
40 INPUT A(1)
50 PRINT " UPPER INDEPENDENT VARIABLE";
60 INPUT A(2)
70 PRINT "DESIRED INDEPENDENT VARIABLE";
80 INPUT X
90 P=(X-A(1))/(A(2)-A(1))
100 PRINT "PRECISION(IN DECIMAL PLACES)";
110 INPUT E
120 IF E=0 THEN 140
130 E=1/(10^E)
140 J=1
150 PRINT
160 GOSUB 470
170 J=2
180 GOSUB 470
190 IF B(1,J)=99999 THEN 300
200 FOR I=2 TO J
210 B(I,J-I+1)=B(I-1,J-I+2)-B(I-1,J-I+1)
220 NEXT I
230 PRINT
240 PRINT J-1;
250 GOSUB 500
260 PRINT " DIFFERENCE =" ;B(J,1)
270 IF B(J,1)<E THEN 300
280 J=J+1
290 IF J<=9 THEN 180
300 Z=0
310 P1=1
320 X=1
330 FOR I=1 TO 8
340 X=X*I

```

```

350 P1=P1*(P-I+1)
360 Z=Z+P1*B(I+1,1)/X
370 NEXT I
380 IF A(2)>A(1) THEN 410
390 Z=B(1,1)-Z
400 GOTO 420
410 Z=B(1,1)+Z
420 PRINT "INTERPOLATION IS TO THE ORDER OF"
430 PRINT J-1;
440 GOSUB 500
450 PRINT " DIFFERENCES ANSWER =" ; Z
460 GOTO 590
469 REM SUBROUTINE TO ENTER TABULAR VALUES
470 PRINT "ENTER TABULAR VALUE AT";A(1)+(J-1)*(A(2)-A(1));
480 INPUT B(1,J)
490 RETURN
499 REM ROUTINE TO PRINT "ST","ND", ETC
500 IF J<>2 THEN 520
510 PRINT "ST";
520 IF J<>3 THEN 540
530 PRINT "ND";
540 IF J<>4 THEN 560
550 PRINT "RD";
560 IF J<5 THEN 580
570 PRINT "TH";
580 RETURN
590 END

```

## References

- Hildebrand, F.B., *Introduction to Numerical Analysis*, 2d. ed., New York: McGraw-Hill, 1974.
- National Bureau of Standards. *Handbook of Mathematical Functions*. Washington, D.C., 1976.
- Phillips, G. M., and Taylor, R. J. *Theory and Applications of Numerical Analysis*. New York: Academic Press, 1973.
- Scheid. *Numerical Analysis*. New York: McGraw-Hill, 1968.

---

## Lagrangian Interpolation

---

This program applies Lagrange's formula for interpolation to a given function. For each succeeding tabular value you enter, the program displays the corresponding difference. Starting with the second difference, you may either calculate the interpolated value or proceed to the next order of difference. If you go on, you have one more option at each succeeding order of difference, and that is to back up and calculate the interpolated value on the previous order of difference. This effectively lets you take an uncommitted look ahead to see whether the next order of difference is smaller than the present one. Thus you need not choose the order of difference beforehand. The program permits three-point through ten-point Lagrangian interpolation.

The program first asks you for the central argument, which is the argument immediately *below* the one you want. It also requests the next higher argument listed in the table, and your desired argument. You must then enter tabular values for the central argument and the arguments on either side of the central argument. The program calls these values  $f_0$ ,  $f_1$  and  $f_{-1}$ , respectively.

At this point the program displays the first and second differences. You have the option of stopping here with three-point interpolation, or going on to the higher orders of difference. If you go on you must enter, one at a time, tabular values  $f_2, f_{-2}, f_3, \dots, f_5$ . As you make each entry, the program displays the next higher difference. You must decide whether to stop and interpolate based on that difference, back up and interpolate on the previous difference, or proceed and enter another tabular value. You can only proceed as far as the ninth difference, since the program calculates at most a ten-point interpolation.

### Program Notes

The program employs the algorithm set forth by Pearson for simplifying the Lagrangian coefficients, thus precluding the need for coefficient tables. The program also disregards the remainder term in Lagrange's formula. Finally, the program does not perform two-point interpolation, since it is of little use.

### Example

Using the following table, determine the sine of 1.0006 radians.

Angle X in radians	Tabular Value Sin X	Name of Tabular Value
0.996	0.83930 30496	$f_{-4}$
0.997	0.83984 62937	$f_{-3}$
0.998	0.84038 86980	$f_{-2}$
0.999	0.84093 02619	$f_{-1}$
1.000	0.84147 09848	$f_0$
1.001	0.84201 08663	$f_1$
1.002	0.84254 99058	$f_2$
1.003	0.84308 81027	$f_3$
1.004	0.84362 54565	$f_4$
1.005	0.84416 19667	$f_5$

Answer: 0.841795015

```
LAGRANGIAN INTERPOLATION
ENTER THE CENTRAL ARGUMENT, NEXT HIGHER
ARGUMENT, AND THE DESIRED ARGUMENT
? 1, 1.001, 1.0006
ENTER F(0)
? .8414709848
ENTER F(1)
```

```

? .8420108663
ENTER F(-1)
? .8409302619
DIFFERENCE # 1 = 5.39882109E-04
DIFFERENCE # 2 = 8.40518624E-07
DO YOU WANT FURTHER DIFFERENCES? (Y/N)
? Y
ENTER F( 2 )
? .8425499058
DIFFERENCE # 3 = 3.25962901E-09
WANT FURTHER DIFFERENCES?
YES(Y), NO(N), ONE LESS(L)
? Y
ENTER F(- 2 )
? .840388698
DIFFERENCE # 4 = 2.32830644E-09
WANT FURTHER DIFFERENCES?
YES(Y), NO(N), ONE LESS(L)
? Y
ENTER F( 3 )
? .8430881027
DIFFERENCE # 5 = 2.32830644E-09
WANT FURTHER DIFFERENCES?
YES(Y), NO(N), ONE LESS(L)
? N
LAGRANGIAN 6 -POINT INTERPOLATION
PRODUCES A VALUE OF .841795015

```

### Practice Problems

1. What is the sine of 1.0001 radians?

Answer: 0.841525014

2. To ten places, the mantissas of the common logarithms of certain arguments are shown below:

Argument	Mantissa
6.1242	0.787 0493 652
6.1243	0.787 0564 565
6.1244	0.787 0635 478
6.1245	0.787 0706 390
6.1246	0.787 0777 300
6.1247	0.787 0848 209

What is the common logarithm mantissa for 6.12449?

Answer: 0.78706993

### Program Listing

```

5 PRINT "LAGRANGIAN INTERPOLATION"
10 DIM D(10,10),E(10),F(10,10),N(4),G(10)
15 G(1)=1
20 F(1,1)=1
29 REM SET UP INITIAL TABLES OF VALUES
30 FOR I=2 TO 10
40 F(I,1)=SGN(I/2-INT(I/2)-.1)
50 FOR J=2 TO I

```

```

60 F(I,J)=(ABS(F(I-1,J-1))+ABS(F(I-1,J)))*SGN(INT((I+J)/2)-(I+J)/2+.1)
70 NEXT J
80 G(I)=G(I-1)*(I-1)
90 NEXT I
100 PRINT "ENTER THE CENTRAL ARGUMENT, NEXT HIGHER"
110 PRINT "ARGUMENT, AND THE DESIRED ARGUMENT"
120 INPUT X1,X2,X3
130 P=(X3-X1)/(X2-X1)
140 IF P<=0 THEN 100
150 IF P>=1 THEN 100
160 PRINT "ENTER F(0)"
170 INPUT D(1,1)
180 PRINT "ENTER F(1)"
190 INPUT D(2,1)
200 PRINT "ENTER F(-1)"
210 INPUT D(3,1)
220 D(1,2)=ABS(D(2,1)-D(1,1))
230 PRINT "DIFFERENCE # 1 =" ; D(1,2)
240 D(2,2)=ABS(D(3,1)-D(1,1))
250 D(1,3)=ABS(D(2,2)-D(1,2))
260 PRINT "DIFFERENCE # 2 =" ; D(1,3)
269 REM GIVE OPERATOR OPTION OF STOPPING NOW OR CONTINUING
270 PRINT "DO YOU WANT FURTHER DIFFERENCES? (Y/N)"
280 INPUT Y$
290 I=3
300 IF Y$="N" THEN 570
310 IF Y$<>"Y" THEN 270
320 I=I+1
329 REM ENTER FOURTH & SUBSEQUENT TABULAR VALUES
330 PRINT "ENTER F(";
340 IF I/2=INT(I/2) THEN 360
350 PRINT "-";
360 PRINT INT(I/2);")"
370 INPUT D(I,1)
380 FOR J=1 TO I-2
390 D(I-J,J+1)=ABS(D(I-J+1,J)-D(I-J-1,J))
400 NEXT J
410 D(1,I)=ABS(D(1,I-1)-D(2,I-1))
420 PRINT "DIFFERENCE #"; I-1; "=" ; D(1,I)
425 IF I=10 THEN 510
430 PRINT "WANT FURTHER DIFFERENCES?"
440 PRINT "YES(Y), NO(N), ONE LESS(L)"
450 INPUT Y$
458 REM OPERATOR MAY STOP NOW, CONTINUE,
459 REM OR GO BACK TO ONE LESS DIFFERENCE
460 IF Y$="N" THEN 570
470 IF Y$="Y" THEN 320
480 IF Y$<>"L" THEN 430
490 I=I-1
500 GOTO 570
509 REM NO MORE THAN NINE DIFFERENCES POSSIBLE
510 PRINT "WANT NINTH DIFFERENCE (N), OR"
520 PRINT "ONLY EIGHTH DIFFERENCE (E)"
530 INPUT Y$
540 IF Y$="N" THEN 570

```

```

550 IF Y#<>"E" THEN 510
560 I=I-1
569 REM LINES 570 TO 630 SET UP VARIABLES USED IN PEARSON'S ALGORITHM
570 N(1)=P^3-P
580 N(2)=N(1)*(P^2-4)
590 N(3)=N(2)*(P^2-9)
600 N(4)=N(3)*(P^2-16)
610 FOR J=1 TO 10
620 E(J)=D(ABS(11-(J*2))+SGN(INT(J/6)),1)/(P+5-J)
630 NEXT J
640 FOR J=1 TO I
650 T=T+E(INT((10-I)/2)+J)*F(I,J)
660 NEXT J
670 IF I/2<>INT(I/2) THEN 690
680 T=T*(P-I/2)
690 PRINT "LAGRANGIAN";I;"-POINT INTERPOLATION"
700 PRINT "PRODUCES A VALUE OF";T*N(INT((I-1)/2))/G(I)
710 END

```

### References

- National Bureau of Standards. *Handbook of Mathematical Functions*. Washington, D. C., 1966.
- Scheid. *Numerical Analysis* (Schaum's series). New York: McGraw-Hill, 1968.
- Vega. *Vollständige Sammlung grösserer logarithmisch-trigonometrischer Tafeln*. 1794. Reprint. New York: Hafner, 1958.

---

## Sums of Powers

---

This program calculates the sum of the Pth powers (up to the 10th powers) of the first N integers. It will also compute the sums of powers which are not the first N integers, but instead a series of higher integers. For example, if you want the sum of squares of the numbers 101 to 1000, subtract the total of the first 100 squares from the total of the first 1000.

### Program Notes

Clearly, a simple algorithm exists for computing the sums of powers: a loop with provision for adding the successive powers obtained. When you want the sum of very lengthy series of integers, the methods in this program are more efficient.

### Example

What is the sum of the first ten 7th powers?

Answer: 18,080,425

```
SUM OF POWERS
THIS PROGRAM COMPUTES THE SUM OF THE P-TH POWERS
(LIMIT: 10) FOR THE FIRST N INTEGERS. ENTER P AND N
? 7,10
THE SUM OF THE 7 TH POWERS OF
THE FIRST 10 INTEGERS IS 18080425
```

### Practice Problems

1. What is the sum of the first 100 5th powers?  
Answer: 1.717083325 times  $10^{11}$ .
2. What is the sum of the first six 10th powers?  
Answer: 71,340,451
3. What is the sum of the squares of the numbers from 101 to 1000?  
Answer: 333,498,500

### Program Listing

```
5 PRINT "SUM OF POWERS"
10 PRINT "THIS PROGRAM COMPUTES THE SUM OF THE P-TH POWERS"
20 PRINT "(LIMIT: 10) FOR THE FIRST N INTEGERS. ENTER P AND N"
30 INPUT P,N
32 P=INT(P)
34 IF P<1 THEN 10
36 IF P>10 THEN 10
39 REM BRANCH TO PROPER POWER
40 ON P GOTO 50,70,90,110,130,150,180,210,240,270
50 S=N*(N+1)/2
```



```

55 PRINT "THE SUM OF THE FIRST POWERS OF"
60 GOTO 380
70 S=N*(N+1)*(2*N+1)/6
75 PRINT "THE SUM OF THE SECOND POWERS OF"
80 GOTO 380
90 S=(N^2)*((N+1)^2)/4
95 PRINT " THE SUM OF THE THIRD POWERS OF"
100 GOTO 380
110 S=N*(N+1)*(2*N+1)*(3*N^2+3*N-1)/30
120 GOTO 370
130 S=(N^2)*((N+1)^2)*(2*N^2+2*N-1)/12
140 GOTO 370
150 S1=(2*N+1)*(3*N^4+6*N^3-3*N+1)
160 S=N*(N+1)*S1/42
170 GOTO 370
180 S1=3*N^4+6*N^3-N^2-4*N+2
190 S=(N^2)*((N+1)^2)*S1/24
200 GOTO 370
210 S1=5*N^6+15*N^5+5*N^4-15*N^3-N^2+9*N-3
220 S=N*(N+1)*(2*N+1)*S1/90
230 GOTO 370
240 S1=2*N^6+6*N^5+N^4-8*N^3+N^2+6*N-3
250 S=(N^2)*((N+1)^2)*S1/20
260 GOTO 370
270 S2=3*N^8+12*N^7+8*N^6-18*N^5
280 S1=S2-10*N^4+24*N^3+2*N^2-15*N+5
290 S=N*(N+1)*(2*N+1)*S1/66
300 GOTO 370
370 PRINT " THE SUM OF THE ";P;"TH POWERS OF"
380 PRINT "THE FIRST ";N;" INTEGERS IS ";S
390 END

```

## Reference

Chemical Rubber Co. *Handbook of Tables for Mathematicians*, 4th ed. Cleveland: 1970.

---

## Factorials

---

This program calculates the factorial of an integer. For the factorial of a small number  $N$  we recursively multiply the integers from 1 through  $N$ . For larger numbers this becomes impractical, and we instead use Stirling's approximation:

$$N! \approx e^{-N} N^N \sqrt{2 N \pi}$$

This has very high accuracy for large  $N$ .

### Program Notes

Some computers have relatively low overflow points. If yours is one, you must modify this program accordingly. The program calculates the factorial of integers greater than 69 (whose factorial exceeds  $10^{100}$ ) using Stirling's approximation, and a technique to make overflow less likely. You can reduce this switch point in line 50.

The program computes Stirling's approximation incrementally. If an interim calculation exceeds  $10^{20}$  or  $10^{30}$ , the program divides it by  $10^{20}$  or  $10^{30}$  (as appropriate). To compensate, it also displays an aggregate multiplier (in words) as part of the answer. You can reduce the chance of overflow by using  $10^{50}$  instead of  $10^{30}$  (hardware permitting). Change the "30" in lines 190, 220, and 230 to "50."

The interim calculations are exponential increments of 5 apart. This may be too quick to avoid overflow. You can change the "5" in lines 150, 160, 170, 180, and twice on line 280 to "4" or some smaller number, but this will of course slow down the program.

Note that for any given computer there is a theoretical limit beyond which overflow cannot be avoided.

### Example

How much is 8!?

Answer: 40320

```
FACTORIALS
ENTER THE NUMBER WHOSE FACTORIAL YOU WANT
? 8
THE FACTORIAL OF 8 IS
40320
TIMES 10 TO THE POWER 0
COMPUTED RECURSIVELY
```

### Practice Problems

1. How much is 100!?

Answer:  $9.3248 \times 10^{157}$

2. What is the factorial of 20?

Answer:  $2.43290201 \times 10^{18}$

3. How much is 141!?

Answer:  $1.897 \times 10^{243}$

## Program Listing

```
5 PRINT "FACTORIALS"
10 PRINT "ENTER THE NUMBER WHOSE FACTORIAL YOU WANT"
20 INPUT N
30 F=1
50 IF N>69 THEN 150
59 REM CALCULATE USING RECURSIVE ALGORITHM
60 FOR I=2 TO N
70 F=F*I
80 IF F<1E+10 THEN 120
100 F=F/(1E+10)
110 J=J+10
120 NEXT I
130 GOTO 300
149 REM CALCULATE USING STIRLING'S APPROXIMATION
150 K=INT(N/5)
160 I=I+5
170 IF I>K*5 THEN 280
180 F=(F*N^5)/EXP(5)
190 IF F>1E+30 THEN 220
200 IF F>1E+20 THEN 250
210 GOTO 160
220 F=F/(1E+30)
230 J=J+30
240 GOTO 190
250 F=F/(1E+20)
260 J=J+20
270 GOTO 190
280 F=(F*N^(N-K*5))/EXP(N-K*5)*SQR(N*6.28318530718)
300 PRINT "THE FACTORIAL OF";N;"IS"
310 PRINT F
320 PRINT "TIMES 10 TO THE POWER";J
330 IF K>0 THEN 360
340 PRINT "COMPUTED RECURSIVELY"
350 GOTO 370
360 PRINT "COMPUTED BY STIRLING'S APPROXIMATION"
370 END
```

## References

- Korn & Korn. *Mathematical Handbook*, 2nd ed. New York: McGraw-Hill, 1968.
- National Bureau of Standards. *Handbook of Mathematical Functions*. Washington, D. C., 1966.

---

## Temperature Conversion

---

Chemists, physicists, and other scientists are constantly involved in taking temperatures in one scale and converting them to other scales. In science, temperatures are commonly recorded and manipulated in five scales: Fahrenheit, Celsius (formerly called Centigrade), Réaumur, Kelvin, and Rankine. This program takes any temperature (above absolute zero) recorded in any scale and converts it into all four of the other scales.

### Example

Convert 98.6° Fahrenheit into the other scales.

```
TEMPERATURE CONVERSION
WHAT IS THE TEMPERATURE WHICH
YOU WISH TO BE CONVERTED?
? 98.6
IN WHAT SCALE WAS THAT RECORDED?
ENTER 1 FOR FAHRENHEIT, 2 FOR
CELSIUS, 3 FOR REAUMUR, 4 FOR
KELVIN, 5 FOR RANKINE
? 1
98.6      DEGREES FAHRENHEIT =
37        DEGREES CELSIUS
29.6      DEGREES REAUMUR
310.1     DEGREES KELVIN
558.18    DEGREES RANKINE
```

### Practice Problems

1. The boiling point of water is 100° Celsius. What is it on the other scales?  
Answer: 212° Fahrenheit, 80° Réaumur, 373.1° Kelvin, 671.58° Rankine.
2. Lonna keeps her hot tub at 104° Fahrenheit. How hot is it on the other scales?  
Answer: 40° Celsius, 32° Réaumur, 313.1° Kelvin, 563.58° Rankine.

### Program Listing

```
5 PRINT "TEMPERATURE CONVERSION"
10 PRINT "WHAT IS THE TEMPERATURE WHICH"
20 PRINT "YOU WISH TO BE CONVERTED?"
30 INPUT T
40 PRINT "IN WHAT SCALE WAS THAT RECORDED?"
50 PRINT "ENTER 1 FOR FAHRENHEIT, 2 FOR"
60 PRINT "CELSIUS, 3 FOR REAUMUR, 4 FOR "
70 PRINT "KELVIN, 5 FOR RANKINE"
80 INPUT S
90 S=INT(S)
100 IF S<1 THEN 40
```

```

110 IF S>5 THEN 40
119 REM BRANCH ON TYPE OF SCALE
120 ON S GOTO 130,170,210,250,290
130 IF T<-459.58 THEN 420
140 T1=T
150 PRINT T," DEGREES FAHRENHEIT = "
160 GOTO 340
170 IF T<-273.1 THEN 420
180 T1=32+T*1.8
190 PRINT T," DEGREES CELSIUS = "
200 GOTO 320
210 IF T<-218.48 THEN 420
220 T1=32+T*2.25
230 PRINT T," DEGREES REAUMUR = "
240 GOTO 320
250 IF T<0 THEN 420
260 T1=32+1.8*(T-273.1)
270 PRINT T," DEGREES KELVIN = "
280 GOTO 320
290 IF T<0 THEN 420
300 T1=T-459.58
310 PRINT T," DEGREES RANKINE = "
320 PRINT T1," DEGREES FAHRENHEIT"
330 IF S=2 THEN 360
340 PRINT 5*(T1-32)/9," DEGREES CELSIUS"
350 IF S=3 THEN 380
360 PRINT 4*(T1-32)/9," DEGREES REAUMUR"
370 IF S=4 THEN 400
380 PRINT 5*(T1-32)/9+273.1," DEGREES KELVIN"
390 IF S=5 THEN 450
400 PRINT T1+459.58," DEGREES RANKINE"
410 GOTO 450
420 PRINT "TEMPERATURE YOU ENTERED DOES NOT "
430 PRINT "EXIST. PLEASE ENTER A NEW ONE"
440 GOTO 10
450 END

```

## Reference

Lange. *Lange's Handbook of Chemistry*, 10th rev. ed. New York: McGraw-Hill, 1967.

---

## Numeric Base Conversion

---

This program will convert numbers between any two bases 2 through 36. The program will continue to convert values from and to the same bases until you enter zero as the value to convert. Then you can enter a new base to convert to, still using the previously entered base to convert from. If you enter zero as the base to convert to, you must enter a new base to convert from. Enter zero at this point to end the program.

### Program Notes

You may convert between a base greater than 36, as long as you define the characters to represent values greater than 35. To do this, add the character(s) you choose between the 'Z' and the closing quotes in line 30. For example, to convert to base 37, we'll represent the number 36 with the character '#'. Change line 30 so that it reads:

```
30 N$="0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ#"
```

Signs, decimal points, and any other characters you enter as part of the value to be converted that are not included in the character representations for the FROM base you selected, are interpreted as zeros wherever they appear.

Note that because the value you enter is first converted to its base 10 value, which is stored in the numeric variable D, accuracy of the output value is limited by the accuracy of your computer. This is also true because of the repeated division used in the conversion process.

You may encounter problems using this program on your computer because of the use of string variables. See the Appendix of this book for information on conversion of programs which use string variables.

### Example

What is the base 16 number ABCD in base 10? What is the base 8 value? What is the base 36 equivalent of the base 10 number 825,062?

Answer: ABCD base 16 is 43,981 base 10. The base 8 value is 125,715. 825,062 base 10 is HOME base 36.

#### NUMERIC BASE CONVERSION

```
FROM BASE (0 TO END)? 16
TO BASE? 10
VALUE? ABCD
ABCD BASE 16 IS 43981 BASE 10
VALUE? 0
TO BASE? 8
VALUE? ABCD
ABCD BASE 16 IS 125715 BASE 8
VALUE? 0
TO BASE? 0
FROM BASE (0 TO END)? 10
TO BASE? 36
VALUE? 825062
825062 BASE 10 IS HOME BASE 36
```

VALUE? 0  
TO BASE? 0  
FROM BASE (0 TO END)? 0

### Practice Problems

1. What is the base 16 representation of the base 10 number 45? What is the base 8 representation?

Answer: 45 base 10 is 2D base 16. 45 base 10 is 55 base 8.

2. What is the base 32 representation of the base 18 number 1G6? What is the base 10 value?

Answer: 1G6 base 18 is JA base 32. 1G6 base 18 is 618 base 10.

### Program Listing

```
10 PRINT "NUMERIC BASE CONVERSION"
20 PRINT
30 N$="0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ"
40 REM -- VARIABLE 'M' IS THE HIGHEST BASE YOU MAY CONVERT FROM / TO
50 M=LEN(N$)
60 PRINT "FROM BASE (0 TO END)";
70 INPUT B1
80 REM -- END PROGRAM?
90 IF B1=0 THEN 450
100 REM -- TEST FOR VALID INPUT BASE
110 IF B1>1 THEN 140
120 PRINT "BASES 2 THROUGH";M;"ONLY. SELECT AGAIN."
130 GOTO 60
140 IF B1>M THEN 120
150 PRINT "TO BASE";
160 INPUT B2
170 IF B2=0 THEN 60
180 REM -- TEST FOR VALID OUTPUT BASE
190 IF B2>1 THEN 220
200 PRINT "BASES 2 THROUGH";M;"ONLY. SELECT AGAIN."
210 GOTO 150
220 IF B2>M THEN 200
230 PRINT "VALUE";
240 INPUT V$
250 IF V$="0" THEN 150
260 REM -- FIRST, CONVERT INPUT VALUE TO BASE 10
270 L=LEN(V$)
280 D=0
290 FOR I=1 TO L
300 FOR J=1 TO B1
310 IF MID$(N$,J,1)<>MID$(V$,I,1) THEN 330
320 D=D+INT((J-1)*(B1^(L-I))+.5)
330 NEXT J
340 NEXT I
350 REM -- NOW CONVERT BASE 10 VALUE TO DESIRED OUTPUT BASE
360 O$=""
370 X=INT(((D/B2)-INT(D/B2))*B2+.5)
380 O$=MID$(N$,X,1) + O$
390 D=INT(D/B2)
```

```
400 IF D>0 THEN 370
410 REM -- OUTPUT THE RESULT
420 PRINT V$;" BASE";B1;"IS ";O$;" BASE";B2
430 REM -- LOOP BACK TO ENTER ANOTHER VALUE
440 GOTO 230
450 END
```



---

## Musical Transposition

---

In music, transposition is the art of playing music in a different key than that in which it was written. Some musicians can transpose by sight or by ear; others have to convert each note from one key into another, laboriously, one-by-one. This program is for those in the latter group. The notes transposed by this program can be used as the roots of harmonies for piano, guitar, etc., as easily as they can be used as single notes.

The program first displays all the keys and key signatures, comprising seven flats through seven sharps, with their identifying numbers. You enter the numbers for the keys from and to which you are transposing. The program then displays each of the 12 possible notes, along with their transposed equivalents.

Note that the program will in all cases print out the correct pitch of the note it is transposing to, and in virtually all cases the correct name as well. However, in those rare cases of some minor keys with multiple accidentals, you may have to supply the alternate name where a double accidental (double sharp or double flat) is called for.

### Example

What do notes in the key of B<sup>b</sup> become when you transpose to the key of G?

Answer:

```
MUSICAL TRANSPOSITION
IN THE FOLLOWING LIST OF KEYS
AND KEY SIGNATURES,
1.  A MAJOR/F-SHARP MINOR-3 SHARPS
2.  B-FLAT MAJOR/G-MINOR-2 FLATS
3.  C-FLAT MAJOR/A-FLAT MINOR-7 FLATS
    B MAJOR/G SHARP MINOR-5 SHARPS
4.  C MAJOR/A MINOR-NO SHARPS OR FLATS
5.  D-FLAT MAJOR/B-FLAT MINOR-5 FLATS
    C-SHARP MAJOR/A-SHARP MINOR-5 SHARPS
6.  D MAJOR/B MINOR-2 SHARPS
7.  E-FLAT MAJOR/C MINOR-3 FLATS
8.  E MAJOR/C-SHARP MINOR-4 SHARPS
9.  F MAJOR/D MINOR-1 FLAT
10. G-FLAT MAJOR/E-FLAT MINOR-6 FLATS
    F-SHARP MAJOR/D-SHARP MINOR-6 SHARPS
11. G MAJOR/E MINOR-1 SHARP
12. A-FLAT MAJOR/F MINOR-4 FLATS
ENTER THE NO. OF THE KEY FROM WHICH
YOU ARE TRANSPOSING, THEN THE NO. OF
THE KEY TO WHICH YOU ARE TRANSPOSING
```

## Practice Problems

1. In the key of G, the first chords of "My Country 'Tis of Thee" are: G, E<sup>m</sup>, C, D, G, E<sup>m</sup>, C, G, B<sup>7</sup>, E<sup>m</sup>. If it is transposed to E, what would these chords be?

Answer: E, C<sup>#m</sup>, A, B, E, C<sup>#m</sup>, A, E, G<sup>#7</sup>, C<sup>#m</sup>.

2. Bach's Fifth Brandenburg Concerto, written in D Major, begins: D, D, F<sup>#</sup>, F<sup>#</sup>, A, A, D, D, C<sup>#</sup>, D, C<sup>#</sup>, B, A, G, F<sup>#</sup>, E. If he had written it in C Major what would these notes have been?

Answer: C, C, E, E, G, G, C, C, B, C, B, A, G, F, E, D.

## Program Listing

```
5 PRINT "MUSICAL TRANSPOSITION"
10 DIM A$(12)
19 REM READ TABLE OF NOTES
20 FOR I=1 TO 12
30 READ A$(I)
40 NEXT I
50 DATA "A","B-FLAT/A-SHARP","B (C-FLAT)","C (B-SHARP)"
60 DATA "D-FLAT/C-SHARP","D","E-FLAT/D-SHARP","E (F-FLAT)"
70 DATA "F (E-SHARP)","G-FLAT/F-SHARP","G","A-FLAT/G-SHARP"
280 PRINT "IN THE FOLLOWING LIST OF KEYS"
290 PRINT "AND KEY SIGNATURES,"
300 PRINT "1. A MAJOR/F-SHARP MINOR-3 SHARPS"
310 PRINT "2. B-FLAT MAJOR/G-MINOR-2 FLATS"
320 PRINT "3. C-FLAT MAJOR/A-FLAT MINOR-7 FLATS"
325 PRINT "   B MAJOR/G SHARP MINOR-5 SHARPS"
330 PRINT "4. C MAJOR/A MINOR-NO SHARPS OR FLATS"
340 PRINT "5. D-FLAT MAJOR/B-FLAT MINOR-5 FLATS"
345 PRINT "   C-SHARP MAJOR/A-SHARP MINOR-5 SHARPS"
350 PRINT "6. D MAJOR/B MINOR-2 SHARPS"
360 PRINT "7. E-FLAT MAJOR/C MINOR-3 FLATS"
370 PRINT "8. E MAJOR/C-SHARP MINOR-4 SHARPS"
380 PRINT "9. F MAJOR/D MINOR-1 FLAT"
390 PRINT "10. G-FLAT MAJOR/E-FLAT MINOR-6 FLATS"
395 PRINT "   F-SHARP MAJOR/D-SHARP MINOR-6 SHARPS"
400 PRINT "11. G MAJOR/E MINOR-1 SHARP"
410 PRINT "12. A-FLAT MAJOR/F MINOR-4 FLATS"
450 PRINT "ENTER THE NO. OF THE KEY FROM WHICH"
460 PRINT "YOU ARE TRANSPOSING, THEN THE NO. OF"
470 PRINT "THE KEY TO WHICH YOU ARE TRANSPOSING"
480 INPUT A,B
500 PRINT
510 IF A>12 THEN 620
520 IF B>12 THEN 620
530 IF A<1 THEN 620
540 IF B<1 THEN 620
610 IF A<>B THEN 710
620 PRINT "ERROR. PLEASE ENTER AGAIN"
630 GO TO 450
710 PRINT "      TRANSPOSITION TABLE"
720 PRINT " TRANSPOSED";TAB(20);"TRANSPOSED"
730 PRINT TAB(4);"FROM";TAB(24);"TO"
740 P=0
749 REM PRINT TABLE
```

```

750 FOR I=1 TO 12
755 D=B-A+I-SGN(INT((B-A+I)/12))*12
757 IF D>0 THEN 760
758 D=12
760 PRINT TAB(1);A$(I);TAB(20);A$(D)
770 P=P+1
780 IF P/3<>INT(P/3) THEN 810
790 PRINT
800 P=0
810 NEXT I
820 PRINT
830 PRINT "DO YOU WANT ANOTHER TRANSPOSITION? (Y/N)"
840 INPUT Y$
850 IF Y$="Y" THEN 280
860 END

```

## References

- Pistan. *Harmony*, 3rd ed. New York: Norton, 1969.
- Priesing and Tecklin. *Language of the Piano*. Boston: Carl Fischer, 1959.

## Appendix

There have been attempts in recent years to establish a standard for Basic. The fact is, no such standard is in widespread use, and this state of affairs is likely to continue indefinitely. Nonetheless, we have tried to use a generalized dialect of Basic that is compatible with many different computers. Still, you may find that some of the conventions we have adopted will not work on your computer. This appendix describes some of the changes you can make to the programs in this book to resolve Basic compatibility problems. If you encounter problems that we do not describe here, you will have to use your own ingenuity to resolve them.

Here in the appendix you will also find suggestions for changing the programs to accommodate different output devices.

We describe each of the specific changes listed below in a general way and illustrate wherever possible with an example taken from the book. You must decide how a suggested change would apply to any particular program, if at all. Therefore, you will need some understanding of Basic programming in order to implement these changes.

### Pausing With Full Display Screen

Many programs have more lines of output than will fit on a typical display screen. This means the first lines of output flash by quickly and scroll off the top of the screen, leaving you with no idea of what they contained. On many computers, you can press a key or a combination of keys to temporarily freeze the display. You can then review and record anything on the display. Subsequently pressing the special key or combination of keys again sets the computer in motion. More program output appears. You may have to freeze the display several times in order to see all the output. The number of times you must freeze the display depends not only on which program you are running, but also on the nature of the problem you present it with. For example, in CBasic you can simultaneously press the CONTROL and S keys to induce this state of suspended animation. On the Radio Shack TRS-80, you do the same thing by simultaneously pressing the SHIFT and @ keys.

Alternatively, you can modify a program so it pauses at one or more points during its output, waiting for the user to cue it to continue. To do this, add the following subroutine to the program, and call the subroutine at suitable intervals during the output phase of the program.

```
5799 REM WAIT FOR OPERATOR CUE
5800 PRINT "ENTER 'C' TO CONTINUE"
5810 INPUT W$
5820 RETURN
```

This technique is used in the Income Averaging program. In programs where some or all of the output occurs inside a loop (e.g., between FOR and NEXT statements), you may not be able to merely place calls to this subroutine between appropriate PRINT statements, as we did in the Income Averaging program on lines 1890, 2010, and 2110. In this case, use the subroutine below, which counts the number of lines displayed since the last pause. Each time you call this subroutine, it increments a counter, and tests to see if the new count exceeds the size of the display. If so, it pauses for the operator cue. Otherwise, it simply returns to the calling point in the program. Therefore, you would insert a call to this subroutine immediately after every PRINT statement that causes a line of output (i.e., a PRINT statement not ending with a comma or semicolon).

```
5797 REM SUBROUTINE CHECKS LINE COUNT
5798 REM WAITS FOR CUE IF DISPLAY IS FULL
5799 REM FIRST INCREMENT AND CHECK LINE COUNT
5800 L9 = L9 + 1
5810 IF L9 < 22 THEN 5850
5819 REM SCREEN IS FULL — —
5820 PRINT "ENTER 'C' TO CONTINUE";
5830 INPUT W$
5839 REM RESET LINE COUNT
5840 L9 = 0
5850 RETURN
```

## String Variables, Functions, and Concatenation

Programs which use simple (non-array) string variables do not dimension them in a DIM statement, in keeping with common practice. If your computer requires it, add a DIM statement to dimension such string variables at the beginning of each program that uses them.

The Home Budgeting program is the only program in this book which uses a string array, C1\$. It is dimensioned on line 20, shown below to have five elements.

```
20 DIM D(12), IO(3,2), CO(4,3), C1(5,5), C1$(5) E0(25,2)
```

If your Basic has different syntax rules for dimensioning string arrays, you must change line 20 of the Home Budgeting program accordingly.

Some programs use the string function MID\$ to specify part of a string. Your Basic may have a different name for this function, like SEG\$. If so, replace each occurrence of MID\$ with the name of the corresponding function in your Basic.

A plus sign (+) is the most common Basic concatenation operator. If your Basic uses a different one, like an ampersand (&), replace each occurrence of the plus sign — where it is used as a concatenation operator — with the correct concatenation operator. Be sure you do not replace any plus signs used for arithmetic addition with the concatenation operator.

## Printer Output

Viewing program output on the display screen is perfectly acceptable when you are using a program as an experimental or investigative tool. But sooner or later, you will probably tire of continually copying program output from the display by hand. The solution, of course, is to direct program output to a printer. The procedure for doing this varies widely from one computer to the next. In some cases, you need only press a key or a combination of keys and all program output appears on both the display and the printer simultaneously. In other cases, you can achieve this same effect (or cause output to appear only on the printer) by entering a sequence of commands just before you run a program.

In many cases, though, you must actually modify each PRINT statement in a program in order to reroute output to the printer. Here again, the procedure for doing this varies widely. Consult your Basic reference manual for specific instructions.

## Replacing DEF FN Statements

Most Basics allow you to define your own single-line function by using a DEF FN statement. You invoke such functions with a FN statement. There is an example of this on lines 20 and 240 of the Future Value of an Investment program shown below. The purpose of this function is to round numeric values to the nearest hundredth.

```
20 DEF FNA(X) = INT(X*100 + .5)/100
240 T=T + FNA(C(J)*(1+R)^(N-J))
```

You can replace such user-defined functions with regular subroutines. The DEF FN statement becomes the subroutine and must be placed outside the normal main line execution path of the program. Each place the function is invoked with an FN statement, you must insert a call to the new subroutine. In the Future Value of an Investment program, you would remove line 20 and replace it with the following subroutine:

```
1000 X = INT(X*3100 + .5)/100
1010 RETURN
```

You would change line 240 as shown below, and add new lines to call the new subroutine.

```
240 X=C(J)*(1+R)^(N-J)
243 GOSUB 1000
245 T=T+X
```

## Changing the Precision of Rounded Values

Many of the programs employ user defined functions to round numeric values to a certain number of decimal places. For example, the Net Present Value program has a function on line 20 which does this:

```
20 DEF FNA(X)=INT(X*100 + .5)/100
```

This function rounds to the nearest hundredth, thus calculating the net present value to the nearest cent. The value 100 which appears twice in the function definition statement shown above determines how many decimal digits there will be (two in this case). To change the number of decimal digits, change both occurrences of the value 100, or whatever value is specified in the program you are considering. For example, the following replacement for line 20 will calculate net present value to the nearest whole dollar:

```
20 DEF FNA(X) = INT(X*1 + .5)/1
```

Or more simply stated:

```
20 DEF FNA(X) = INT(X + .5)
```

## Frequency of Compounding Interest

Several of these programs base their computations on interest compounded annually. This is acceptable in most cases. But you can have the calculations compound interest more frequently. Perhaps the easiest way to do this is to convert the annual interest rate to the effective interest rate, based on the number of compounding periods per year. Then enter this effective rate when the program asks for an interest rate. The general formula for this is:

$$E = \left( 1 + \frac{I}{N} \right)^{NY}$$

where E is the effective interest rate, I is the annual interest rate expressed as a decimal fraction, N is the number of compounding periods per year, and Y is the number of years. The formula for continuous compounding is:

$$E = e^{IY}$$

where E is the effective interest rate,  $e$  is 2.718281828 . . . (the base of natural logarithms), I is the nominal interest rate, and Y is the number of years.

Of course, you can change a program to accept the nominal interest rate and convert it automatically to the effective interest rate. The program would have to ask for the number of compounding periods per year in order to make the conversion. Alternatively, you could restate the interest compounding calculation in the program so that it compounds at the desired frequency. For example, this calculation occurs in the Future Value of an Investment program on line 240. If you restate line 240 as shown below, the program will compute the future value of an investment at growth rate R, compounded continuously.

```
240 T=T + FNA(C(J)*EXP(R*(N-J)))
```



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