Number Tree
Factorial trees are generated by using the leftmost digit of a factorial value as the top of the tree. Subsequent digits are pulled out of the actual value, from left to right, in groups of slightly increasing size. The groups are placed below, and in a symmetrical position to, the cornerstone digit building up a tree shape. This diagram represents the factorial of 32. The value can be read more simply when the numbers are placed in groups of three, as shown

These procedures depend on the fact that numbers in LOGO are treated as words. By using them, we can now define a procedure, F:

```
TO F :X
  PRINT IMPLODE FACT EXPLODE :X
END
```

Which will calculate the factorial of 13 in response to the input: F13.

The result of this calculation — 6227020800 — is a little hard to read as such. It is more usual to insert commas (6,227,020,800), which makes it easier to understand. The following procedures divide the word up into groups of three digits and insert commas.

```
TO ADDCOMMAS :X
  IF ((COUNT :X)<4) THEN OUTPUT :X
  OUTPUT (WORD ADDCOMMAS BUTTHREE :X ",
  LASTTHREE :X)
END

TO BUTTHREE :X
  OUTPUT BUTLAST BUTLAST BUTLAST :X
END

TO LASTTHREE :X
  OUTPUT (WORD (LAST BUTLAST BUTLAST :X)
  (LAST BUTLAST :X) (LAST :X))
END
```

We must also modify F to incorporate these procedures:

```
TO F :X
  PRINT ADDCOMMAS IMPLODE FACT EXPLODE :X
END
```

Using F to print out the first 20 factorials gives some idea of how quickly factorials grow in size (the results are given in the table).

Having obtained the factorials of a range of numbers, we can begin to 'play around' with our results. An American mathematician, for example, once had the brilliant idea of printing out large factorial numbers as trees on the Christmas cards he sent to his friends. Not many factorials have the right number of digits to be printed as trees, but the following procedures will work if it is possible to do so:

```
TO TREE :L
  TREE1 1 :L
END

TO TREE1 :NO :L
  IF EMPTY? :L THEN STOP
  REPEAT ROUND (20 — :NO / 2) [PRINT1 SPACE]
  LINEPRINT :NO :L
  TREE1 :NO + 2 PRUNE :NO :L
END

TO SPACE
  OUTPUT CHAR 32
END

TO LINEPRINT :NO :L
  IF :NO = 0 THEN PRINT "STOP
  PRINT1 FIRST :L
  LINEPRINT :NO — 1 BUTFIRST :L
END

TO PRUNE :NO :L
  IF :NO=0 THEN OUTPUT :L
  OUTPUT PRUNE :NO -1 BUTFIRST :L
END
```

Once again, our controlling procedure must be modified:

```
TO F :X
  TREE IMPLODE FACT EXPLODE :X
END
```

The diagram shows 32! written out as a tree. If you are interested in exploring these factorial trees further, you might like to know that there are only three numbers less than 32 whose factorials can be written as trees. The next larger suitable factorial is 59!

Logo Flavours

For all LCSI versions use:

EMPTYP for EMPTY?
AND for ALLOF
TYPE for PRINT1

There is also a different IF syntax. For example:

IF :CARRY = 0 [OUTPUT []][OUTPUT (LIST :CARRY)]

In place of QUOTIENT :X :Y use DIV :X :Y on the Spectrum and ROUND (:X / :Y) on the Atari.

Use SE for SENTENCE on the Atari.