can therefore apply 'invalid input' conditions (X) to the other six. Table 1 is the truth table. Now, we must consider each of the 10 outputs separately. It would seem that the Boolean expression for each output must be made up of four terms $\mathrm{W}, \mathrm{X}, \mathrm{Y}$ and Z , as in Table 2.
Some simplification is possible, however. As the invalid input conditions are the same for each of the 10 outputs we can link terms together to simplify them. Below is a four-variable Karnaugh map with the six invalid input conditions marked.


If we now consider each output in turn we may find that some simplification can be made. Let us take, for example, the output for the digit 3 . Placing the Boolean expression on the $k$-map we can see that a loop can be drawn that includes it.


Thus the expression for the output can be simplified to X.Y.Z. Taking output 9 as a second example:


A loop can be drawn that uses three of the invalid input conditions and represents the Boolean expression W.Z. Many of the other output terms simplify in a similar way. You may wish to check for yourselves that the simplified output terms are those shown in Table 3.

All that now remains to be done is to construct the circuit diagram from the 10 Boolean expressions. As each input is used both in its normal form and its negative form it is easiest to construct the circuit from eight parallel lines representing these terms. Each of the 10 outputs can then be formed by branching off from the relevant lines and using AND gates. The completed decoder circuit diagram is given below.

## Exercise 6

1) A three-input encoder is to be designed to create an output of 1 for inputs $011,101,110$, or 111. The output should be zero for all other input combinations.
a) Draw the truth table for the encoder
b) Produce a Boolean expression for theoutputand simplify it
c) Draw the encoder circuit

Table 1

| Inputs |  |  |  | $\begin{array}{\|c} B C D \\ - \text { Digit } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| W | I |  |  |  |
| 0 | , | 0 |  | 0 |
| 0 | 0 | 0 |  | 1 |
| 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 1 |  | 3 |
| 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 |  | 5 |
| 0 | 1 | 1 |  | 6 |
| 0 | 1 | 1 |  | 7 |
| 1 | 0 | 0 |  | 8 |
| 1 | 0 | 0 |  | 9 |
|  | 0 | 1 |  | x |
| 1. | 0 | 1 |  | $x$ |
| 1 | 1 | 0 |  | $x$ |
| 1 | 1 | 0 | 1 | $x$ |
|  | 1 | 1 |  | $x$ |
|  |  |  |  |  |

Table 2

| BCD |  |
| :---: | :---: |
| Digit | Boolean |
| 0 | Expression |
| 1 | W.X.Y.Z |
| 2 | W.X.Y.Z |
| 3 | W.X.Y.Z |
| 4 | W.X.Y.Z |
| 4 | W.X.Y.Z |
| 5 | W.X.Y.Z |
| 6 | W.X.Y.Z |
| 7 | W.X.Y. |
| 8 | W.X.Y.Z |
| 9 | W.X.Y.Z |


| $\begin{array}{\|l\|} \hline \text { BCD } \\ \text { Digit } \end{array}$ | Boolean Expression |
| :---: | :---: |
| 0 | W. X.Y. 2 |
| 1 | W.X.Y.Z |
| 2 | X.Y.Z |
| 3 | X.Y. Z |
| 4 | X.Y. Z |
| 5 | X.Y.Z |
| 6 | X.Y.Z |
| 7 | x.y.z |
| 8 | W.Z |
| 9 | W. 2 |

