If this output is negated then we will get:

$$
\overline{\overline{\mathrm{A} . \mathrm{B}}}=\mathrm{A} \cdot \mathrm{~B}
$$

So the circuit will be:


OR Gates: Just as chaining two NAND gates together is equivalent to an AND gate, so if we chain two NOR gates together we obtain a circuit that is equivalent to an OR gate:


The required output from an OR gate is $\mathrm{A}+\mathrm{B}$. Using the rules of Boolean algebra, this can be manipulated into a NAND form:

$$
\begin{aligned}
\mathrm{A}+\mathrm{B} & =\overline{\overline{\mathrm{A}}}+\overline{\overline{\mathrm{B}}} \\
& =\overline{\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}}}
\end{aligned}
$$

and consequently the corresponding circuit using NAND gates is:


If we wish to construct a circuit using only NAND or NOR elements then we may still follow the simplification methods we have already met, but first we must manipulate the final Boolean expression into a form that is suitable. For circuits incorporating NAND gates, we use the rules of Boolean algebra to create an expression that consists of groups of ANDs connected by ORs, and use de Morgan's theorem repeatedly until the expression is completely in NAND form. For circuits in the NOR form, we employ similar rules, as the example will show. To demonstrate how these rules are used let's look again at the

Exclusive OR (XOR) gate (see page 47). The output from an XOR gate can be defined by the expression $\mathrm{C}=\overline{\mathrm{A} . \mathrm{B}} .(\mathrm{A}+\mathrm{B})$.

Let us take this expression and convert it so that a circuit for the XOR gate may be constructed solely from NAND gates. First of all, let's manipulate the expression so that we obtain groups of ANDs connected by ORs.
$\mathrm{C}=\overline{\bar{A} \cdot \mathrm{~B}} \cdot(\mathrm{~A}+\mathrm{B})$

$$
\begin{aligned}
& ==(\overline{\mathrm{A} \cdot \mathrm{~B}} \cdot \mathrm{~A})+(\overline{\mathrm{A} \cdot \mathrm{~B}} \cdot \mathrm{~B})(\text { multiply out brackets) } \\
& =(\overline{\overline{\mathrm{A} \cdot \bar{B}} \cdot \mathrm{~A}}) \cdot(\overline{\mathrm{A} \cdot \bar{B} \cdot \mathrm{~B}})(\text { de Morgan's theorem })
\end{aligned}
$$

When drawing the circuit from a complicated expression such as the one above, it is best to start from the output and work backwards to the inputs. Try following this circuit diagram from output to input to see how it was constructed.


For the NOR form, we must again start with the original simplified expression for the XOR gate and manipulate it into groups of ORs connected by ANDs. This first step can be done by using de Morgan's theorem on the left hand part:

$$
\begin{aligned}
\mathrm{C} & =\overline{\mathrm{A} \cdot \mathrm{~B}} \cdot(\mathrm{~A}+\mathrm{B}) \\
& =(\overline{\mathrm{A}}+\overline{\mathrm{B}}) \cdot(\mathrm{A}+\mathrm{B}) \\
& =(\overline{\overline{\mathrm{A}}+\overline{\mathrm{B}}})+(\overline{\mathrm{A}+\mathrm{B}})
\end{aligned}
$$

Converting this expression into a circuit diagram is again best done by starting at the output and working backwards.


Answers To Exercise 6 On Page 147

1a) | Inputs |  |  | Output |
| :---: | :---: | :---: | :---: |
| A | B | $\mathbf{C}$ | $\mathbf{P}$ |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

b) The Boolean expression for $P$ is:
$P=\bar{A} \cdot B \cdot C+A \cdot B \cdot C+A \cdot B \cdot \bar{C}+A \cdot B . C$
Simplification may be achieved by using a Karnaugh map:


Thus $\mathrm{P}=\mathrm{A} \cdot \mathrm{B}+\mathrm{A} \cdot \mathrm{C}+\mathrm{B} \cdot \mathrm{C}$
c) The circuit is:


