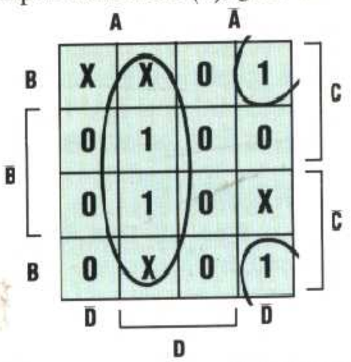
$$S = \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.B.C.\overline{D} + A.\overline{B}.\overline{C}.D + A.\overline{B}.C.D$$

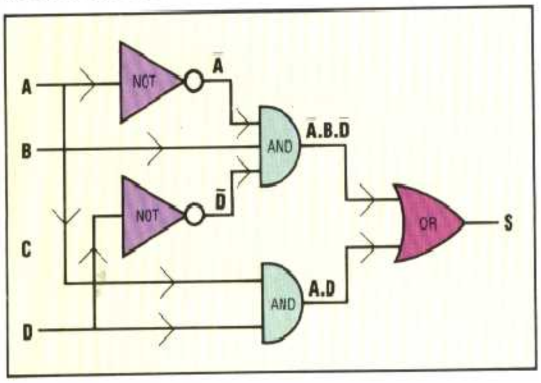
Drawing these on a k-map, together with the 'invalid input' conditions (X), gives us:




From this k-map we can see that the expression reduces to:

$$A.D + \overline{A}.B.\overline{D}$$

And thus, our 'thirty day month' signal circuit can be constructed:




**Example 2: Odd Numbers**

Given that the numbers 0 to 15 can be coded by four binary digits (0000 to 1111), we are asked to design a circuit that will accept the four bit code as an input and output a 1 if the output represents an odd number greater than two.

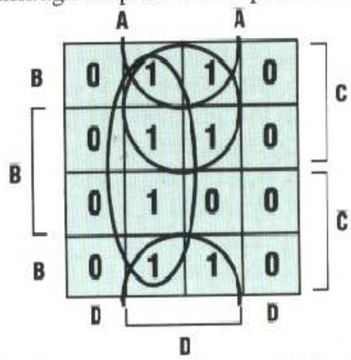
The first thing we must do is set up a truth table for all the possible conditions:

| DECIMAL | INPUTS | | | | OUTPUT |
|---|---|---|---|---|---|
| NUMBER | A | B | C | D | S |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 1 |

From this truth table we form the following Boolean algebra expression, for all the conditions where S is true (= 1):

$$S = \overline{A}.\overline{B}.C.D + \overline{A}.B.\overline{C}.D + \overline{A}.B.C.D + A.\overline{B}.\overline{C}.D + A.\overline{B}.C.D + A.B.\overline{C}.D + A.B.C.D$$

The Karnaugh map for this expression is:



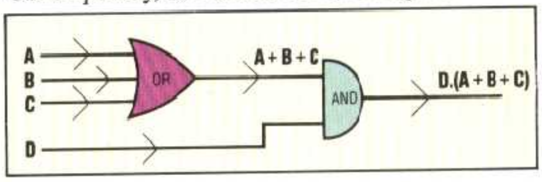

From the k-map, three groups of fours can be isolated, represented by this expression:

$$S = A.D + C.D + B.D$$

and this can be further simplified using the distributive law to get:

$$S = D.(A + B + C)$$

Consequently, the circuit can be designed:




In the next instalment, we will review the more important aspects of the Logic course so far, and provide a comprehensive set of review exercises.

**Exercise 5**

**1)** Simplify the following Boolean expressions, using Karnaugh maps:

**a)** $A.B.C + A.\overline{B}.\overline{C} + \overline{A}.\overline{C} + \overline{A}.B.C + A.B.\overline{C}$

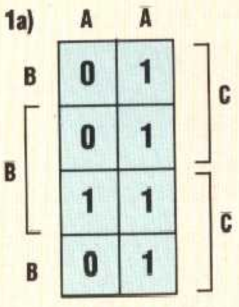
**b)** $\overline{B + C} + B.\overline{C} + A.C$

**c)** $A.\overline{B}.D + \overline{A}.D + A.B.C.D + A.B.\overline{C} + \overline{A}.B.\overline{C}.\overline{D}$

**2)** A circuit is to be designed that will accept the binary representations of the whole numbers between 0 and 7 inclusive. The circuit is to give an output if the number input is odd or if it is a multiple of three (i.e. 3 or 6). By drawing a truth table and obtaining a simplified expression, draw a logic circuit that will carry out this function.

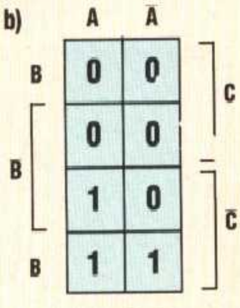
**Answers to Exercise 4 on Page 93**

**1a)**




$\overline{A} + \overline{B}.\overline{C}$

**b)**




$(A + B).\overline{C}$