MAP READING

Karnaugh maps are extremely useful devices in the simplification of logic circuits. Whilst not entirely replacing the algebraic simplifications that we have investigated previously (see page 47), they do dispense with much of the effort involved in the factorisation of complicated Boolean algebra expressions.

Karnaugh maps (also known as *k-maps*) are really extensions of the Venn diagrams that we encountered earlier in the course (see page 46), which allow us to represent logical expressions pictorially. A k-map takes on slightly different forms depending on the number of different letters (or variables) there are in the expression to be simplified, but is most useful for expressions containing two, three or four variables.

Two Variables: Each square of a two variable k-map (there are $2^2 = 4$ squares) represents an ANDing function, as shown in this diagram:

	A	$\overline{A}$
B	A.B	$\overline{A}$.B
$\overline{B}$	A.$\overline{B}$	$\overline{A}$.$\overline{B}$

To represent the expression $AB + A\overline{B}$ as a k-map, we place ones in the relevant squares:

	A	$\overline{A}$
B	1	0
$\overline{B}$	1	0

Here are three further examples, representing the expressions $\overline{A}\,\overline{B}$, $AB + \overline{A}\,\overline{B}$, and $AB + \overline{A}B + A\overline{B}$ respectively:

	A	$\overline{A}$
B	0	0
$\overline{B}$	0	1

	A	$\overline{A}$
B	1	0
$\overline{B}$	0	1

	A	$\overline{A}$
B	1	1
$\overline{B}$	1	0

Three Variables: In this case, the number of squares increases by a factor of two ($2^3 = 8$ squares). The basic three variable k-map is:

	A	$\overline{A}$	
B	A.B.C	$\overline{A}$.B.C	C
$\overline{B}$	A.$\overline{B}$.C	$\overline{A}$.$\overline{B}$.C	
	A.$\overline{B}$.$\overline{C}$	$\overline{A}$.$\overline{B}$.$\overline{C}$	$\overline{C}$
B	A.B.$\overline{C}$	$\overline{A}$.B.$\overline{C}$	

Here are two expressions, $AC + \overline{A}B\overline{C}$ and $AB + \overline{A}C$, represented as k-maps:

	A	$\overline{A}$	
B	1	0	C
$\overline{B}$	1	0	
	0	0	$\overline{C}$
B	0	1	

	A	$\overline{A}$	
B	1	1	C
$\overline{B}$	0	1	
	0	0	$\overline{C}$
B	1	0	

$$ABC+A\overline{B}C+\overline{A}B\overline{C}$$
$$=AC(B+\overline{B})+\overline{A}B\overline{C}$$
$$=AC+\overline{A}B\overline{C}$$

$$ABC+AB\overline{C}+\overline{A}BC+\overline{A}\,\overline{B}C$$
$$=AB(C+\overline{C})+\overline{A}C(B+\overline{B})$$
$$=AB+\overline{A}C$$

Notice that both expressions have been simplified using the Boolean law that a set A ORed with its negation ($\overline{A}$) produces 1 — the Universal or Identity set.

Four Variables: When we start dealing in four variables, the maps get more complex (they have $2^4 = 16$ squares), but nevertheless are quite simple to interpret according to the basic grid:

	A		$\overline{A}$		
B	A.B.C.$\overline{D}$	A.B.C.D	$\overline{A}$.B.C.D	$\overline{A}$.B.C.$\overline{D}$	C
$\overline{B}$	A.$\overline{B}$.C.$\overline{D}$	A.$\overline{B}$.C.D	$\overline{A}$.$\overline{B}$.C.D	$\overline{A}$.$\overline{B}$.C.$\overline{D}$	
	A.$\overline{B}$.$\overline{C}$.$\overline{D}$	A.$\overline{B}$.$\overline{C}$.D	$\overline{A}$.$\overline{B}$.$\overline{C}$.D	$\overline{A}$.$\overline{B}$.$\overline{C}$.$\overline{D}$	$\overline{C}$
B	A.B.$\overline{C}$.$\overline{D}$	A.B.$\overline{C}$.D	$\overline{A}$.B.$\overline{C}$.D	$\overline{A}$.B.$\overline{C}$.$\overline{D}$	
	$\overline{D}$	D		$\overline{D}$	