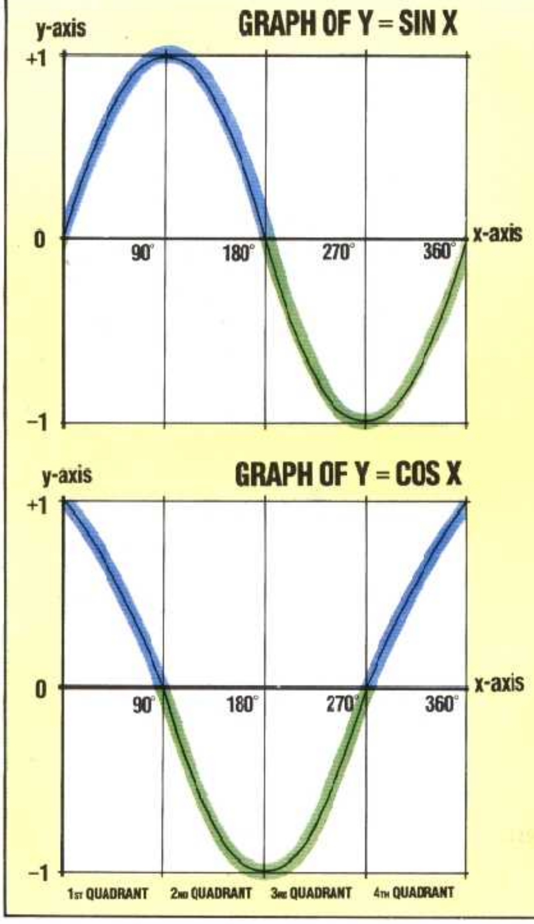
wonder what relevance to the real world trigonometry could possibly have. The answer lies in the fact that 'trig' (as it is called) is the link between Euclidean geometry, which deals with the manipulation of points, lines and curves, and algebra, which allows mathematical solutions to be arrived at by manipulating variables with unknown values. Take the parabola, for example. This is a curve with many useful properties: and all sorts of things can be discovered about these properties using graph paper, a protractor, a ruler and a pencil. Far more useful, though, is the realisation that the curve can be represented by the algebraic formula: $y = x^2$

This simple formula allows us to calculate values for any point on the curve without resorting to actually drawing it. Problems that can be dealt with algebraically lend themselves to computer solutions far more easily than problems that require drawn graphs or figures, so the advantage of trigonometrical solutions should be immediately apparent.

The objective of the first two articles of this series is to give a brief overview of trigonometric concepts and to see how these can be quite easily coded into BASIC programs. Let's start by taking a look at the basic trigonometrical functions of *cosine* and *sine*.



One Wave

The familiar 'sine wave' pattern is produced by graphing values of the sine function for a complete circle. Along the x-axis are the angles from 0° to 360° and the y-axis represents the range of sine values for those angles. Notice that all the sine values lie between plus and minus one. The four sections of the graph correspond to the quadrants shown in the circle opposite; sine is positive (shown in blue) for the first two quadrants and negative (green) afterwards. Graphing cosine achieves similar results. The cosine curve is actually the same shape as the sine curve but it is shifted along the x-axis by 90 degrees

CALCULATING THE RATIO

Cosine and sine are two ways of relating the ratio of the sides of right-angled triangles. Cosine and sine are also directly related to the circle. Any right-angled triangle can be drawn to fit inside a circle, called the 'unit' circle. It is given this name because it has a radius of one 'unit' — the actual measurement doesn't matter because it's the ratios that count. The illustration shows a line that has been 'rotated' by 35°. The starting position for a rotation, by convention, is the horizontal axis, and the direction of rotation is anti-clockwise. The horizontal axis is called the *x-axis* and the angle of rotation is called *theta*, written using the Greek letter Θ. If a line is dropped from the circumference to the x-axis, you get a right-angled triangle.

The cosine of Θ is defined as the ratio of the length of the *adjacent* side of the triangle (the part lying along the x-axis) to the *hypotenuse* (the radius of the unit circle). If we take, as an example, a circle with a radius of 15mm, and measure precisely the adjacent side of the triangle inscribed within it, we would get a result of approximately 12.28728mm. Dividing this value by 15 gives a result of 0.819152, which is the cosine value for 35°. If you have a calculator with a COS key (you can also use a set of cosine tables if you know how), try entering 35 COS. You should get a result of 0.819152044. This ratio holds true whatever size of the unit circle the right-angled triangle is inscribed in. Whether the radius of the circle is one inch, one mile or one light year, the side of the triangle lying along the x-axis will always be approximately 0.82 of the radius' value.

We could also draw in other values of Θ in the unit circle illustrated. It can be seen that for any value of Θ the value of cos Θ (cos is the usual abbreviation for cosine, and is pronounced 'koz') will never be greater than 1 nor less than 0. For values of Θ greater than 90°, cos Θ will, however, have a negative value. This is because cos Θ involves the value of the x co-ordinate (the corresponding position on the x-axis of the point where the hypotenuse intersects the circumference of the unit circle). Mathematical convention assumes that the point of origin of the rotation is at 0 on the x-axis. Any points to the left of that will have negative values. For the same reason, the cosine of angles between 180° and 270° will also be negative, but the cosine of angles greater than 270° and up to 360° will again become positive.

The *sine* function of an angle is very similar to the cosine except that it involves values on the y-axis (the vertical axis). If Θ is 0, the adjacent side of the triangle will be equal in length to the hypotenuse. The co-ordinate of P on the x-axis will always be 1 (because $1/1 = 1$), but the co-ordinate on the y-axis will be 0. For all values of Θ up to 90°, sin Θ ('sin', pronounced 'sign', is the usual abbreviation of 'sine') will range from 0 to 1. In the second quadrant of the circle, sin Θ will also be positive, but will decrease from a maximum of 1 down to 0 as Θ approaches 180°. All values of Θ greater than 180° and up to (but not including) 360° will be negative.