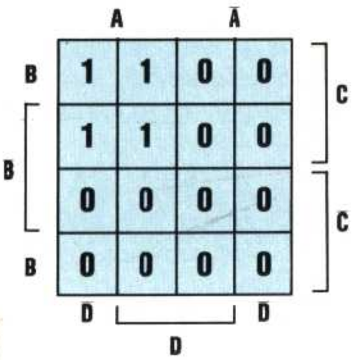
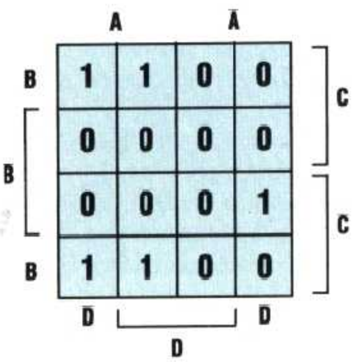
Here is one k-map with an accompanying simplification:



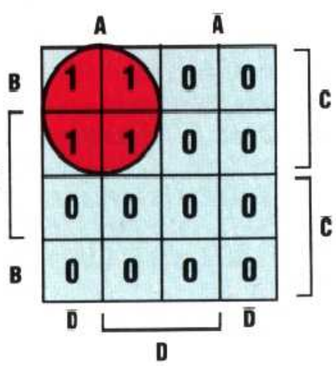
$ABC\overline{D} + ABCD + A\overline{B}C\overline{D} + A\overline{B}CD$
$= ABC(\overline{D} + D) + A\overline{B}C(\overline{D} + D)$
$= ABC + A\overline{B}C$
$= AC(B + \overline{B})$
$= AC$

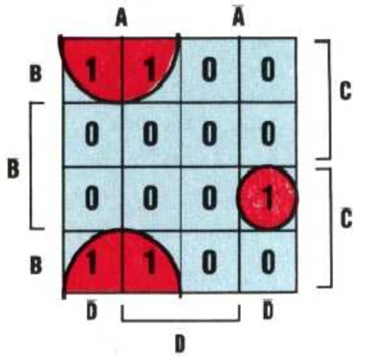
Here is another example:



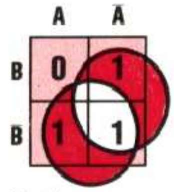
$ABC\overline{D} + ABCD + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D}$
$+ AB\overline{C}D$
$= ABC(\overline{D} + D) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}(\overline{D} + D)$
$= ABC + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}$
$= AB(C + \overline{C}) + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$= AB + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$

If we look closely at the arrangement of the ones in these two k-maps we can discover patterns in them. In the first example, all the squares with AC in their expressions have 1 in them. In the second example this is the case with all the AB squares. This suggests that an easier way of simplifying Boolean expressions is simply to inspect a k-map. Consider these:





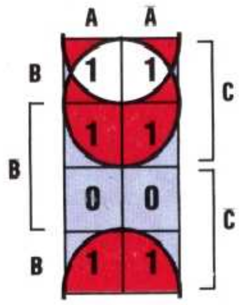
With a little practice it is possible to pick out groups of 2, 4 or 8 ones to form simpler terms. For example, let's consider this expression: $\overline{A}B + A\overline{B} + \overline{A}\,\overline{B}$.



Using a two variable k-map, we can pick out two groups of ones. One group represents all the NOT(B) cases and the other represents all the NOT(A) cases, so we can simplify the expression to $\overline{A} + \overline{B}$. This expression can be further simplified, using de Morgan's law, to: $\overline{A.B}$. Is it possible to arrive at the conclusion more directly by inspecting the k-map?

A more difficult example involves a three variable expression:

$ABC + A\overline{B}C + \overline{A}BC + AB\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$



The group of four ones at the top of the k-map represent all the possible cases in which C is true. The top and bottom rows of the map represent all the possible cases in which B is true. Hence the simplified expression is: $B + C$.

In the next instalment of the course, we will continue our investigation into the use of Karnaugh maps in the simplification of Boolean expressions involving four variables. Then we will show how k-maps are used in the process of circuit design. This will bring together all the aspects of the course discussed so far.

Exercise 4:
Draw up three-variable k-maps to simplify the following Boolean expressions:

a) $\overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{B}.\overline{C} + \overline{A}.B.\overline{C}$

b) $A.\overline{B}.\overline{C} + \overline{A}.B.\overline{C} + A.B.\overline{C}$