# Sweet Sixteen 

## Computers work in binary notation, since they can cope with only two states. So why do programmers sometimes use hexadecimal, which is base 16 ?

It is easy to see why the binary system is used internally in computers - the representation of numbers using just ones and zeros lends itself well to the on/off electrical signals of the computer. It is also easy to see why the decimal system is almost universally used in human society - we happen to have ten fingers and so it has become our natural number base.

But why should hexadecimal (base 16) have any use? The reason is that hexadecimal numbers convert to and from binary numbers much more easily than decimal numbers do. Since computers 'understand' binary numbers, why don't programmers program in binary digits? There are two reasons. Firstly, binary numbers are much longer than their base 10 or base 16 equivalents. For example, the number 356 in decimal is 0000000101100100 in binary. Secondly, it is much easier to make mistakes using numbers comprising only ones and zeros.

The hexadecimal system uses 16 different digits, including zero, so that the largest number that can be represented in the 'ones' column is equivalent to decimal 15. Mathematicians could have invented six new symbols for the numbers between ten and 15 , but the convention is to use the first six letters of the alphabet for the extra six digits. When we count by adding ones, we use the standard numeric digits together with the alphabetic digits until we reach $F-$ the hexadecimal equivalent of 15 . After that we run out of symbols and have tostart a new column, the 'sixteens' column. Here, too, we can use the available symbols up to F . When both the 'sixteens' column and the 'ones' column are full, adding one means we have to start a third column, the 'two-hundred-and-fifty-sixes' column. The panel shows some decimal numbers with their binary and hexadecimal equivalents.

The number 65,535 represented in hexadecimal is FFFF, so one of the advantages of the hexadecimal system is immediately apparent. A number that takes five digits to represent in decimal and 16 to represent in binary needs only four digits in hexadecimal. Apart from this compactness, there is a more important advantage. Four binary digits can be represented by exactly one hexadecimal digit. This makes converting from binary to hexadecimal and vice versa relatively straightforward.

To convert a binary number into its hexadecimal equivalent, divide the binary number into groups of four digits, starting from the right,

and convert them to hexadecimal a group at a time. Here are some examples:

|  |  | $\begin{aligned} & 1110 \\ = & \\ = & 233 \end{aligned}$ | $\begin{aligned} & 1001 \\ & 9 \end{aligned}$ | binary hex decimal |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 1111 \\ = & F \\ = & 3980 \end{aligned}$ | $\begin{aligned} & 1000 \\ & 8 \end{aligned}$ | $\begin{aligned} & 1100 \\ & C \end{aligned}$ | binary hex decima |
| $\begin{aligned} & 0111 \\ & =7 \\ & =32,301 \end{aligned}$ | $1110$ <br> E | $\begin{aligned} & 0010 \\ & 2 \end{aligned}$ | $\begin{aligned} & 1101 \\ & D \end{aligned}$ | binary hex decima |

When would you actually want to use hexadecimal notation? Although high-level languages such as BASIC do not require you to use hexadecimal (or binary for that matter), low level languages such as machine code and assembly language do.

Numbers written in hexadecimal are often written with an H after them to distinguish them from decimal numbers. Thus, 256 (decimal) would be written 100 H and pronounced 'one zero zero hex'.

We won't be making much practical use of these hexadecimal numbers in The Home Computer Course. However, you may well come across the term in appendices to your computer's instruction manual. Now, you can at least work out their decimal or binary equivalent.

## The Hexadecimal Abacus

An abacus made for the hexadecimal system would have 15 beads on each rod. On the rightmostrod, beads can be flicked down one at a time until the last bead, $F$, is reached. Hexadecimal is a base-16 number system and letters of the alphabet are used for the numbers higher than 9. As shown on the middle rod, A corresponds to decimal $10, B$ to 11, C to 12, D to 13, E to 14 and Fto 15. Alter the 'number' F, we run out of digits and have to move over to the next column, the ' 16 's' column.

The ' 1 's' column in the illustration shows a count of 9 beads. The ' 16 's' column shows a count of $F$ and the - 256 's' column shows a count of 2 . This is read as '2F9 hex'. It is equivalent to the decimal number $761(256 \times 2+16 \times 15$ $+1 \times 9$ )

| Binary |  | Decimal |
| :---: | :---: | :---: |
| 00000001 | 1 | Hex |
| 00000010 | 2 | 2 |
| 00000011 | 3 | 3 |
| 00000100 | 4 | 4 |
| 00000101 | 5 | 5 |
| 00000110 | 6 | 6 |
| 00000111 | 7 | 7 |
| 00001000 | 8 | 8 |
| 00001001 | 9 | 9 |
| 00001010 | 10 | A |
| 00001011 | 11 | B |
| 00001100 | 12 | C |
| 00001101 | 13 | D |
| 00001110 | 14 | E |
| 00001111 | 15 | F |
| 00010000 | 16 | 10 |
| 00010001 | 17 | 11 |
| 00010010 | 18 | 12 |
| 00010011 | 19 | 13 |
| 00010100 | 20 | 14 |
| 00010101 | 21 | 15 |

